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Application of Exponential-Gamma Distribution in Modeling Queuing Data

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ABSTRACT

There are many events in daily life where a queue is formed. Queuing theory is the study of waiting lines and it is very crucial in analyzing the procedure of queuing in daily life of human being. Queuing theory applies not only in day to day life but also in sequence of computer programming, networks, medical field, banking sectors etc. Researchers have applied many statistical distributions in analyzing a queuing data. In this study, we apply a new distribution named Exponential-Gamma distribution in fitting a data on waiting time of bank customers before service is been rendered. We compared the adequacy and performance of the results with other existing statistical distributions. The result shows that the Exponential-Gamma distribution is adequate and also performed better than the existing distributions.

KEYWORDS: *Exponential-Gamma distribution, Queuing theory, AIC, BIC, Networks*

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INRODUCTION

There are many events in daily life where a queue is formed. Queuing theory is the study of waiting lines and it is very crucial in analyzing the procedure of queuing in daily life of human being. Queuing theory applies not only in day to day activities but also in sequence of computer programming, networks, medical field, banking sectors etc. A queuing theory called a random service theory is one of the issues in mathematics so that the existing techniques in the queuing theory have substantial importance in solving mathematical problems and analyzing different systems [1].

Statistical distributions are very crucial in describing and predicting real life occurrence. Although, many distributions have been developed and there are always rooms for developing distributions which are more flexible and capable of handling real world application. Lately, studies have shown that some real life data cannot be analyzed adequately by existing distributions. At times, it may be discovered to follow distributions of some combined form of two or more random variables with known probability distributions. In light of their adequacies, variety in usage and performance, statistical distributions have received a numerous attentions from various researchers such as; [2],[3],[4] and [5]. Therefore this study aimed to examine the adequacy and performance of the new Exponential-Gamma distribution to other exiting probability distributions using the data on waiting time of bank customers before service is being rendered, using the model selection criteria like the Akaike information criterion (AIC), Bayesian information criterion (BIC) and the log likelihood function (l).

METHODS

The Exponential-Gamma distribution was developed by [6] and its pdf is defined as

$$f(x;\alpha,\lambda)=\frac{\lambda^{\alpha+1}x^{\alpha-1}e^{-2\lambda x}}{\Gamma(\alpha)}, x,\lambda,\alpha>0 \qquad (1)$$

With the mean and variance;

$$\mu=\frac{\alpha}{2^{\alpha+1}} \qquad (2)$$

and
$$V(x)=\frac{\alpha\left(\alpha 2^{\alpha}-\lambda\alpha+2^{\alpha}\right)}{\lambda\left(2^{2(\alpha+1)}\right)} \qquad (3)$$

The cumulative distribution function is defined as

$$F(x)=\frac{\lambda\gamma(\alpha,x)}{2^{\alpha}\Gamma(\alpha)} \qquad (4)$$

The survival function for the distribution defined by $S(x)=1-F(x)$ was obtained as;

$$S(x)=1-\frac{\lambda\gamma(\alpha,x)}{2^{\alpha}\Gamma(\alpha)} \qquad (5)$$

While the corresponding hazard function defined by $h(x)=\frac{f(x)}{S(x)}$ was obtained as;

$$h(x)=\frac{\lambda^{\alpha+1}x^{\alpha-1}e^{-2\lambda x}2^{\alpha}}{2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)} \qquad (6)$$

The cumulative hazard function for distribution defined by $H(x)=W(F(x))=-\log(1-F(x))\equiv\int_0^x h(x)dx$ and was obtained as;

$$H(x)=\frac{\lambda\gamma(\alpha,x)}{2^{\alpha}\Gamma(\alpha)-\lambda\gamma(\alpha,x)} \qquad (7)$$

A. Maximum Likelihood Estimator

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from Exponential-Gamma distribution. Then the likelihood function is given by;

$$L(\alpha,\lambda;x)=\left(\frac{\lambda^{\alpha+1}}{\Gamma(\alpha)}\right)^n\prod_{i=1}^{n}x_i^{\alpha-1}\exp\left(-2\lambda\sum_{i=1}^{n}x_i\right) \qquad (8)$$

by taking logarithm of (9), we find the log likelihood function as;

$$\log(L)=\alpha n\log\lambda+n\log\lambda-n\log\Gamma(\alpha)+(\alpha-1)\sum_{i=1}^{n}\log x_i-2\lambda\sum_{i=1}^{n}x_i \qquad (9)$$

Therefore, the MLE which maximizes (9) must satisfy the following normal equations;

$$\frac{\partial\log L}{\partial\alpha}=n\log\lambda-\frac{n\Gamma'(\alpha)}{\Gamma(\alpha)}+\sum_{i=1}^{n}\log x_i \qquad (10)$$

$$\frac{\partial\log L}{\partial\lambda}=\frac{\alpha n}{\lambda}+\frac{n}{\lambda}-2\sum_{i=1}^{n}x_i \qquad (11)$$

The solution of the non-linear system of equations is obtained by differentiating (9) with respect to (α,λ) gives the maximum likelihood estimates of the model parameters. The estimates of the parameters can be obtained by solving (10) and (11) numerically as it cannot be done analytically. The numerical solution can also be obtained directly by using python software using the data sets.

In this study, we applied the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and the log likelihood function (l) to compare the new developed Exponential-Gamma distribution with the existing probability distributions such as the Exponential and the Gamma distributions.

The AIC is defined by;

$$AIC=2k-2In(\hat{\ell}) \qquad (12)$$

where k is the number of the estimated parameter in the model

$\hat{\ell}$ is the maximized value of the model.

The BIC is defined by;

$$In(n)k-2In(\hat{\ell}) \qquad (13)$$

where k is the number of the estimated parameter in the model

n is the number of observations

$\hat{\ell}$ is the maximized value of the model.

The approach in (12) and (13) above is used when comparing the performance of different distributions to determine the best fit model. To select the appropriate model by considering the number parameters and maximum likelihood function; the AIC, BIC and likelihood function are examined; consequently an acceptable model has smaller AIC and BIC value while the log likelihood value is expected to be greater. The Python software was used for the comparison of the performance of the Exponential-Gamma, Exponential and Gamma distributions.

RESULTS

The analysis of the data set was carried out by using python This data has been previously used by [7], [8] and [9]. It represents the waiting time (measured in min) of 100 bank customers before service is being rendered. The data is as follows:

Table1

0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1, 3.2,3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7,4.7, 4.8, 4.9, 4.9, 5.0, 5.3, 5.5, 5.7, 5.7, 6.1, 6.2, 6.2, 6.2,6.3, 6.7, 6.9, 7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6,8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11.0,11.0, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13.0, 13.1, 13.3, 13.6,13.7, 13.9, 14.1, 15.4, 15.4, 17.3, 17.3, 18.1, 18.2, 18.4, 18.9, 19.0, 19.9,20.6, 21.3, 21.4, 21.9, 23.0, 27.0, 31.6, 33.1, 38.5.

Table2: Summary of the data

Parameters	Values
n	100
Min	0.800
Max	38.500
Mean	9.877
Variance	52.37411
Skewness	2.540292
Kurtosis	1.472765

The results from the table 2 above indicated that the distribution of the data is skewed to the right with skewness 2.540292. This shows that Exponential-Gamma has the ability to fit a right skewed data. Also, it was observed that the kurtosis is 1.472765 which is lesser than 3. This implies that the distribution has shorter and lighter tails with a light peakedness when compared to that of the Normal distribution.

Table3: Estimates and performance of the distributions (bank customers)

Distribution	Parameters	log likelihood(l)	AIC	BIC
Exponential-Gamma	$\hat{\alpha} = 1.7$ $\hat{\lambda} = 0.4$	-12.5	30.9	38.7
Exponential	$\hat{\alpha} = 0.8$	-320.6	645.2	650.3591
Gamma	$\hat{\alpha} = 1.7$ $\hat{\lambda} = 0.5$	-316.7	639.5	647.3

The estimates of the parameters, log-likelihood, Akaike information criterion (AIC) and Bayesian information criterion (BIC) for the data on waiting time of 100 bank customers before service is being rendered is presented in Table 3 It was observed that Exponential-Gamma provides a better fit as compared to Exponential and Gamma distributions since it has highest value of log-likelihood (l) and the lowest value of Akaike information criterion (AIC) and Bayesian information criterion (BIC). Hence, the Exponential-Gamma distribution performed better than other distribution compared.

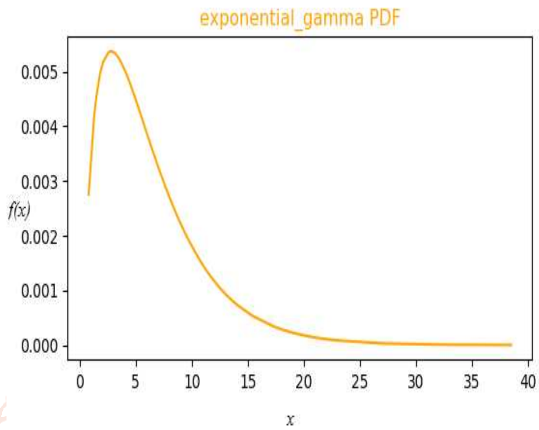

Figure:1 Exponential-Gamma distribution pdf plot for the data sets

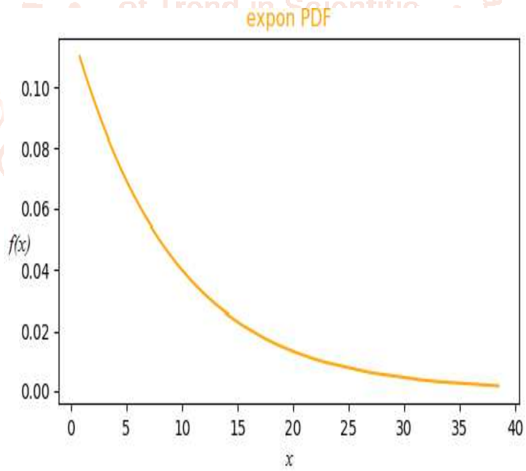

Figure:2 Exponential distribution pdf plot for the data sets

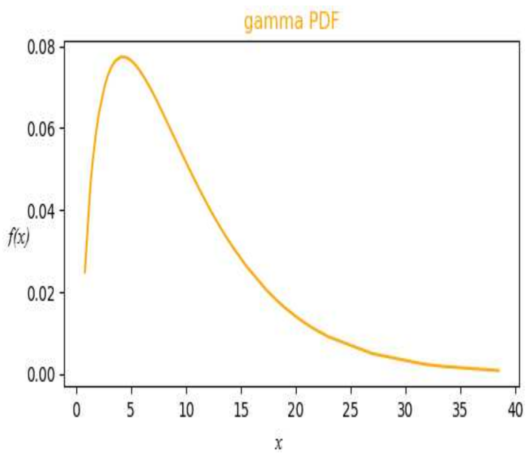

Figure 3: Gamma distribution pdf plot for the data sets

CONCLUSION

There are many events in daily life where a queue is formed. Queuing theory is the study of waiting lines and it is very crucial in analyzing the procedure of queuing in daily life of human being. In this study we apply the new Exponential-Gamma in modeling queuing data on bank customers waiting time before been serviced. We compared the adequacy and performance of the results of new Exponential-Gamma distribution with the exiting probability distributions. The new Exponential-Gamma distribution is adequate and performed better than other distribution compared. It also fits the data better than other existing distributions. Therefore, for higher precision in analyzing queuing data, the use of Exponential-Gamma distribution is highly recommended in various fields where analysis of queuing data is crucial.

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