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RP-47: Formulation of a Class of Standard Congruence of Higher Degree Modulo a Power of an Odd Prime Integer

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ABSTRACT

In this paper, the author considered a problem: "Formulation of solutions of a class of standard congruence of higher degree modulo a power of an odd prime integer". It is studied rigorously and the solutions of the congruence are formulated. Illustrating suitable examples, the formula established is tested and verified true. No earlier formulation is found in the literature of mathematics for such congruence. Author has tried his best to find a formula for the solutions of the problem under consideration and succeed. It is the merit of the paper.

KEYWORDS: *Binomial expansion, Composite modulus, Congruence of higher Degree, Formulation*

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INTRODUCTION

The author already has formulated a lot of standard quadratic, cubic and bi-quadratic congruence of prime and composite modulus and also formulated the solutions of the congruence of any higher degree, successfully [1], [2], [3], [4]. Though the author has tried his best to formulate different congruence of prime as well as **composite modulus**, even some congruence remains to formulate. Here the author wished to consider one such standard congruence of higher degree, yet not formulated. This is the standard congruence of higher degree of the type:
$x^n \equiv p^n \ (mod\ p^m)$, p being an odd prime integer; n, m are any positive integers.

PROBLEM-STATEMENT

In this study, the problem under consideration is
"To formulate the solutions of the standard congruence of higher degree of the type:
$x^n \equiv p^n \ (mod\ p^m), m > n;$

p being an odd prime & n, m are any positive integers in three cases:
Case-I: when n is an even positive integer,
Case-II: when n is an odd positive integer.
Case-III: when $m \leq n$.

LITERATURE REVIEW

In the literature of mathematics, no formulation is found for the solutions of the said congruence. Only the standard quadratic congruence of prime and composite modulus are discussed [5], [6], [7]. No proper & suitable method is found to find the solutions in the literature of mathematics. The author already got published some papers on the formulation of the standard congruence of composite modulus such as:
$x^2 \equiv p^2 \ (mod\ b.p^m)$ [8]
$x^3 \equiv p^3 \ (mod\ b.p^m)$, [9]
$x^4 \equiv p^4 \ (mod\ b.p^m)$[10].

Generalising these papers, the author wishes to formulate the solutions of the standard congruence of higher degree of the type $x^n \equiv p^n \ (mod\ p^m), m > n$.

ANALYSIS & RESULT (Formulation)

Case-I: Let n be an even positive integer and consider the congruence under consideration: $x^n \equiv p^n \ (mod\ p^m);\ m > n$.

Also consider that $x \equiv p^{m-n+1}k \pm p \ (mod\ p^m)$
Then, $x^n \equiv (p^{m-n+1}k \pm p)^n \ (\text{mod } p^m)$

$$\equiv (p^{m-n+1}k)^n \pm n(p^{m-n+1}k)^{n-1}.p + \frac{n.(n-1)}{2}(p^{m-n+1}k)^{n-2}.p^2 \pm \cdots \ldots \ldots \ldots ..$$

$\pm n.p^{m-n+1}k.p^{n-1} + p^n \ (mod\ p^m)$

$\equiv p^n + p^m(\dots\dots) \ (mod\ p^m)$

$\equiv p^n \ (mod\ p^m)$

Thus, it can be said that $x \equiv p^{m-n+1}k \pm p \ (mod\ p^m)$ are the two solutions of the said congruence for every value of k.

But, if one has the value $k = p^{n-1}$, then $x \equiv p^{m-n+1}k \pm p \ (mod\ p^m)$ reduces to

$x \equiv p^{m-n+1}.p^{n-1} \pm p \ (mod\ p^m)$

$\equiv p^m \pm p \ (mod\ p^m)$

$\equiv\ 0 \pm p \ (mod\ p^m)\ \equiv\ \pm p \ (mod\ p^m)$, which are the same solutions as for k=0.

Similarly, it can also be seen that for the next higher values of k, the corresponding solutions repeats as for k=1, 2……..$(p^{n-1} - 1)$.

Therefore, the required solutions are given by

$$\equiv p^{m-n+1}k \pm p \ (mod\ p^m); k = 0, 1, 2, \dots \dots \dots \dots \dots, (p^{n-1} - 1).$$

It is also seen that for a single value of k, the congruence has two solutions and here k has p^{n-1} values. Thus, total number of incongruent solutions are definitely $2p^{n-1}$ for even n.

Case-II: Let n be an odd positive integer and consider the congruence under consideration: $x^n \equiv p^n \ (mod\ p^m);\ m > n$.

Also consider that $x \equiv p^{m-n+1}k \pm p \ (mod\ p^m)$

Then, $x^n \equiv (p^{m-n+1}k \pm p)^{\ n}$ (mod p^m)

$\equiv \pm p^n \ (mod\ p^m)$ as seen above.

Thus, $x \equiv p^{m-n+1}k \pm p \ (mod\ p^m)$ cannot be the solutions.

But if, $x \equiv p^{m-n+1}k + p \ (mod\ p^m)$, then , $x^n \equiv p^n \ (mod\ p^m)$.

Thus, it can be said that $x \equiv p^{m-n+1}k + p \ (mod\ p^m)$ is a solution of the congruence.

If one has the value $k = p^{n-1}$, then $x \equiv p^{m-n+1}k + p \ (mod\ p^m)$ reduces to $x \equiv p^{m-n+1}.p^{n-1} + p \ (mod\ p^m)$

$\equiv p^m + p \ (mod\ p^m)$

$\equiv\ 0 + p \ (mod\ p^m)\ \equiv\ p \ (mod\ p^m)$,which is the same solution as for k=0.

Similarly, it can also be seen that for next higher values of k, the corresponding solutions repeats as for k=1, 2 ……… $(p^{n-1} - 1)$.

Therefore, the required solutions are given by

$x \equiv p^{m-n+1}k + p \ (mod\ p^m)$; k $= 1, 2, \dots\dots., (p^{n-1} - 1)$.

It is also seen that for a single value of k, the congruence has one solution and here k has p^{n-1} values. Thus, total number of incongruent solutions are definitely p^{n-1} for odd n.

Case-III: Let $m \le n$.

If $m \le n$, then the congruence under consideration reduces to: $x^n \equiv p^n \ (mod\ p^n)$

and then the Equivalent congruence is: $x^n \equiv 0 \ (mod\ p^n)$.

For $x \equiv pt \ (mod\ p^n), t\ being\ an\ integer,$

$x^n \equiv (pt)^{\ n} \ (mod\ p^n)$

$\equiv p^{\ n}t^n \ (mod\ p^n)$

$\equiv 0 \ (mod\ p^n)$,

and thus the solutions are given by:

$x \equiv pt \ (mod\ p^n), t\ being\ an\ integer$.

ILLUSTRATIONS

Consider the congruence $x^6 \equiv 729 \ (mod\ 2187)$.

It can be written as $x^6 \equiv 3^6 \ (mod\ 3^7) with\ p = 3, m = 7, n = 6$.

It is of the type $x^n \equiv p^n \ (mod\ p^m); m > n; n = 6, an\ even\ positive\ integer$.

It has $2p^{n-1} = 2p^5 = 2.243 = 486\ solutions$ and the solutions are given by

$x \equiv p^{m-n+1}k \pm p \ (mod\ p^m); k = 0, 1, 2, \dots\dots (p^5 - 1)$.

$\equiv 3^{7-6+1}k \pm 3 \ (mod\ 3^7)$

$\equiv 3^2k \pm 3 \ (mod\ 3^7)$

$\equiv 9k \pm 3 \ (mod\ 2187); k = 0, 1, 2, 3, 4, \dots, 10, \dots\dots, 100, \dots\dots, (243 - 1)$.

$\equiv 0 \pm 3; 9 \pm 3; 18 \pm 3; 27 \pm 3; 36 \pm 3; \dots; 90 \pm 3; \dots\dots; 900 \pm 3; \dots\dots$

$\equiv 3, 2178; 6, 12; 15,21; 24, 30; 33, 39; \dots; 87, 93; \dots\dots; 897, 903; \dots\dots (mod\ 2187)$.

These are the required 486 solutions.

Consider the congruence $x^5 \equiv 243 \ (mod\ 729)$.

It can be written as: $x^5 \equiv 3^5 (mod 3^6)$.

It is of the type:

$x^n \equiv p^n (mod p^m); m > n\ with\ n\ odd\ positive\ integer$.

As n is an odd positive integer, the congruence has only $p^4 = 81\ solutions$.

Solutions are given by

$x \equiv p^{m-n+1}k + p \ (mod\ p); k = 0, 1, 2, 3, 4, \dots\dots (p^{n-1} - 1$.

$\equiv 3^{6-4}k + 3 \ (mod\ 3^6); k = 0, 1, 2, 3, 4, \dots\dots\dots 80$.

$\equiv 3^2k + 3 \ (mod\ 729)$

$\equiv 9k + 3 \ (mod\ 729)$

$\equiv 0 + 3; 9 + 3; 18 + 3; 27 + 3; 36 + 3; 45 + 3; 54 + 3;\ \dots\dots 711 + 3;$

$720 + 3 \ (mod\ 729)$.

$\equiv 3, 12, 21, 30, 39, 48, 57, \dots\dots\dots\dots 714, 723 \ (mod\ 729)$.

These are the required 81 solutions.

Consider the congruence $x^5 \equiv 0 \ (mod\ 81))$.

It can be written as: $x^5 \equiv 0 \ (mod\ 3^4)$.

It is of the type: $x^n \equiv 0 \ (mod\ p^n)$.

The solutions are given by

$x \equiv pt \ (mod\ p^n), t\ being\ an\ integer$.

Here $p = 3$.

Therefore the solutions are $x \equiv 3t \ \ (mod\ 3^4); t = 1, 2, 3 \dots\dots$

$\equiv 3, 6, 9, 12 \dots\dots., (mod\ 81)$.

i.e. all multiples of three are the solutions.

CONCLUSION

Thus, it can be concluded that the standard congruence of higher degree of the type $x^n \equiv p^n \ (mod\ p^m); m > n$, has $2p^{n-1}$ incongruent solutions, if n is an even positive integer, given by $x \equiv p^{m-n+1}k \pm p \ (mod\ p^m); k = 0, 1, 2, \ldots\ldots (p^{n-1} - 1)$.

But the congruence has only p^{n-1} incongruent solutions, if n is an odd positive integer, given by $x \equiv p^{m-n+1}k + p \ (mod\ p^m); k = 0, 1, 2, \ldots\ldots (p^{n-1} - 1)$.

If $m \leq n$, then the congruence reduces to the equivalent congruence $x^n \equiv 0 \ (mod\ p^n)$ and the solutions are given by: $x \equiv pt \ (mod\ p^n), t\ being\ an\ integer$.

MERIT OF THE PAPER

In this paper, a special class of standard congruence of higher degree modulo an odd prime power integer, is formulated in three cases separately. First time, a formula is established. This is the merit of the paper.

REFERENCE

[1] Roy B M, *Formulation of solutions of a special standard quadratic congruence of composite modulus- an even integer multiple of a power of an odd positive integer*, International Journal of Advance Research, Ideas, and Innovations in Technology (IJARIIT), ISSN: 2454-132X, Vol-5, Issue-5, Sep-Oct-2019.

[2] Roy B M, *Formulation of standard quadratic congruence of composite modulus- a product of twin primes*, International Journal of Trend in Scientific Research and development (IJTSRD), ISSN: 2456-6470, Vol-03, Issue-05, Jul-Aug-2019.

[3] Roy B M, Formulation of Solutions of a Class of Solvable Standard Quadratic Congruence of Composite Modulus- a Prime Positive Integer Multiple of eight, International Journal of Mathematics Trends and Technology (IJMTT), INNS: 2231-5373, Vol-61, Issue-4, Sep-18, Pages: 227-229.

[4] Roy B M, Formulation of Solutions of a Class of Solvable Standard Quadratic Congruence of Composite Modulus- a Prime Positive Integer Multiple of four, International Journal of Science & Engineering Development Research (IJSDR), ISSN: 2455-2631, Vol-3 , Issue-11, Sep-18, Pages: 236-238.

[5] Zuckerman at el, An Introduction to The Theory of Numbers, fifth edition, Wiley student edition, INDIA.

[6] Thomas Koshy, 2009, "*Elementary Number Theory with Applications*", 2/e Indian print, Academic Press, ISBN: 978-81-312-1859-4.

[7] Roy B M, "*Discrete Mathematics & Number Theory*", 1/e, Jan. 2016, page No- 88, Das Ganu Prakashan, Nagpur.

[8] Roy B M, *Formulation of Solutions of a Class of Solvable Standard Quadratic Congruence of Composite Modulus*, International Journal of Scientific Research and Engineering Development (IJSRED), ISSN: 2581-7175, Vol-02, Issue-6, Nov-Dec-2019.

[9] Roy B M, *Formulation of Solutions of a Class of Solvable Standard Cubic Congruence of Composite Modulus- an Integer Multiple of power of an odd prime*, International Journal of Science & Engineering Development Research (IJSDR), ISSN: 2455-2631, Vol-4 , Issue-11, November-19, Pages: 236-238.

[10] Roy B M, *Formulation of a special class of standard Bi-quadratic congruence of composite modulus-an integer multiple of power of an odd prime*, International Journal of Engineering, Science and Mathematics (IJESM),ISSN: 2320-0294, Vol-08, Issue-12, December-19, Pages-01 to 04.