International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 4 Issue 2, February 2020 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

Two Types of Novel Discrete-Time Chaotic Systems

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ABSTRACT

In this paper, two types of one-dimensional discrete-time systems are firstly proposed and the chaos behaviors are numerically discussed. Based on the time-domain approach, an invariant set and equilibrium points of such discrete-time systems are presented. Besides, the stability of equilibrium points will be analyzed in detail. Finally, Lyapunov exponent plots as well as state response and Fourier amplitudes of the proposed discrete-time systems are given to verify and demonstrate the chaos behaviors.

KEYWORDS: *Novel chaotic systems, discrete-time systems, Lyapunov exponent*

How to cite this paper: Yeong-Jeu Sun "Two Types of Novel Discrete-Time Chaotic Systems" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-4 | Issue-2, February 2020, pp.1-4, URL: www.ijtsrd.com/papers/ijtsrd29853.pdf

IJTSRD29853

1. INRODUCTION

In recent years, various types of chaotic systems have been widely explored and excavated. As we know, since chaotic system is highly sensitive to initial conditions and the output behaves like a random signal, several kinds of chaotic systems have been widely applied in various applications such as master-slave chaotic systems, secure communication, ecological systems, biological systems, system identification, and chemical reactions; see, for instance, [1-10] and the references therein.

In this paper, two new types of chaotic systems will be firstly proposed. Both of invariant set and equilibrium points of such chaotic systems will be investigated and presented. Finally, various numerical methods will be adopted to verify the chaotic behavior of the proposed two novel discrete-time systems.

This paper is organized as follows. The problem formulation and main result are presented in Section 2. Some numerical simulations are given in Section 3 to illustrate the main result. Finally, conclusion is made in Section 4.

2. PROBLEM FORMULATION AND MAIN RESULTS

Let us consider the following two types of one-dimensional discrete-time systems

The first type of Sun's discrete-time systems:

$$\begin{aligned} x(k+1) &= 0.5\times[-(b+d)x(k)+d+c\ln[2-x(k)] \\ &\quad -a\ln[x(k)+1]]\times\operatorname{sgn}[x(k)-0.5] \\ &\quad +0.5\times[(b-d)x(k)+d+a\ln[x(k)+1] \\ &\quad +c\ln[2-x(k)]],\quad k\in Z^+, \end{aligned} \tag{1a}$$

where

$$a=\frac{2-b}{2\ln(1.5)},\quad c=\frac{2-d}{2\ln(1.5)} \tag{1b}$$

with

$$0\le x(0)\le 1,\quad 1<b<2,\quad 1<d<2, \tag{1c}$$

and

$$\operatorname{sgn}(z):=\begin{cases}1, & z\ge 0\\ -1, & z<0.\end{cases} \tag{1d}$$

The second type of Sun's discrete-time systems:

$$\begin{aligned} x(k+1) &= 0.5\times[(b+d)(x(k)-0.5)+a\ln[x(k)+0.5] \\ &\quad -c\ln[1.5-x(k)]]\times\operatorname{sgn}[x(k)-0.5] \\ &\quad +0.5\times[(b-d)(x(k)-0.5)+a\ln[x(k)+0.5] \\ &\quad +c\ln[1.5-x(k)]],\quad k\in Z^+, \end{aligned} \tag{2a}$$

where

$$a=\frac{2-b}{2\ln(1.5)},\quad c=\frac{2-d}{2\ln(1.5)} \tag{2b}$$

with

$$0\le x(0)\le 1,\quad 1<b<2,\quad 1<d<2. \tag{2c}$$

Before presenting the main result, let us introduce a definition which will be used in the main theorem.

Definition 1: A set $S\subseteq R$ is an invariant set for the discrete system (1) if $x(0)\in S$ implies $x(k)\in S$, for all $k\in N$.

Now we present the first main result.

Theorem 1: The set of $[0,1]$ is an invariant set for the discrete systems (1) and (2).

Proof. It is easy to see that if $x(k) \in [0,1]$ implies $x(k+1) \in [0,1], \forall k \in Z^+$. Consequently, we conclude that if $x(0) \in [0,1]$ implies $x(k) \in [0,1], \forall k \in N$. This completes the proof. ϒ

Now we present the second main results.

Theorem 2: The set of equilibrium points of the system (1) and (2) are given by $\{0, \bar{x}\}$ and $\{0, \hat{x}\}$, respectively, where $\bar{x}$ and $\hat{x}$ satisfy the following equations

$$\bar{x} = d(1-\bar{x}) + \frac{2-d}{2\ln(1.5)}\ln(2-\bar{x}),$$

$$\hat{x} = d(0.5-\hat{x}) + \frac{2-d}{2\ln(1.5)}\ln(1.5-\hat{x}).$$

Furthermore, all of above equilibrium points are unstable.

Proof. (i) (Analysis of the system (1))

Let us define

$$f(x) := 0.5 \times [-(b+d)x + d + c\ln(2-x) - a\ln(x+1)] \times \mathrm{sgn}(x-0.5) + 0.5 \times [(b-d)x + d + a\ln(x+1) + c\ln(2+x)].$$

From the equation of $x = f(x)$, it results that $x = 0$ and $\bar{x}$, with

$$\bar{x} = d(1-\bar{x}) + c\ln(2-\bar{x}) = d(1-\bar{x}) + \frac{2-d}{2\ln(1.5)}\ln(2-\bar{x}).$$

In addition, it is easy to see that

$$f'(0) = a + b > 1 \text{ and } f'(\bar{x}) = -d - \frac{c}{2-\bar{x}} < -1.$$

This implies that both of equilibrium points 0

(ii) (Analysis of the system (2))

Let us define

$$g(x) := 0.5 \times [(b+d)(x-0.5) + a\ln(x+0.5) - c\ln(1.5-x)] \times \mathrm{sgn}(x-0.5) + 0.5 \times [(b-d)(x-0.5) + a\ln(x+0.5) + c\ln(1.5-x)].$$

From the equation of $x = g(x)$, it can be readily obtained that $x = 1$ and $\hat{x}$, with

$$\hat{x} = d(0.5-\hat{x}) + c\ln(1.5-\hat{x}) = d(0.5-\hat{x}) + \frac{2-d}{2\ln(1.5)}\ln(1.5-\hat{x}).$$

Meanwhile, one has $g'(\hat{x}) = -d - \frac{c}{1.5-\hat{x}} < -1$ and $g'(1) = b + \frac{2}{3}a > 1$. It follows that both of equilibrium points $\hat{x}$ and 1 are unstable. This completes the proof. ϒ

3. NUMERICAL SIMULATIONS

Lyapunov exponent plots of the discrete-time systems of (1), with $x(0) = 0.25$, is depicted in Figure 1. Time response of $x(k)$ and Fourier amplitudes for the nonlinear system (1), with $x(0) = 0.25$ and $(b,d) = (1.8, 1.7)$, are depicted in Figure 2 and Figure 3, respectively. Besides, the Lyapunov exponent plots of the nonlinear systems of (2), with $x(0) = 0.35$, is depicted in Figure 4. Time response of $x(k)$ and Fourier amplitudes for the discrete-time system (2), with $x(0) = 0.35$ and $(b,d) = (1.6, 1.9)$, are depicted in Figure 5 and Figure 6, respectively. The simulation graphs show that both of systems (1) and (2) have chaotic behavior. This is due to the fact that all of Lyapunov exponent are larger than one.

4. CONCLUSION

In this paper, two types of Sun's one-dimensional discrete-time systems are firstly proposed and the chaos behaviors are numerically discussed. Based on the time-domain approach, an invariant set and equilibrium points of such discrete-time systems have been presented. Besides, the stability of equilibrium points has been analyzed in detail. Finally, Lyapunov exponent plots as well as state response and Fourier amplitudes for the proposed discrete-time systems have been given to verify and demonstrate the chaos behaviors.

ACKNOWLEDGEMENT

The author thanks the Ministry of Science and Technology of Republic of China for supporting this work under grant MOST 107-2221-E-214-030. Furthermore, the author is grateful to Chair Professor Jer-Guang Hsieh for the useful comments.

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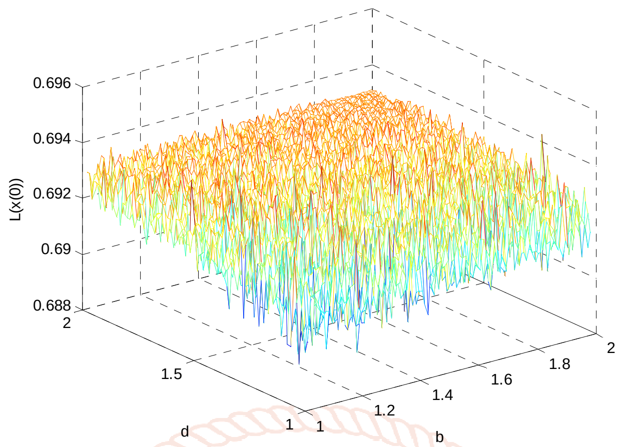

Figure 1: Lyapunov exponents of the system (1). Initial value $x(0)=0.25$, sample size 5×10^3 points, and initial 10^4 points discarded.

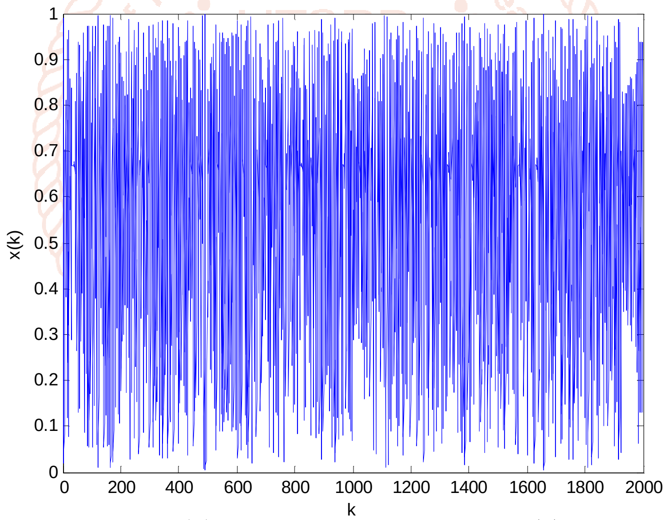

Figure 2: The time response of $x(k)$ for the system (1), with $x(0)=0.25$ and $(b,d)=(1.8,1.7)$.

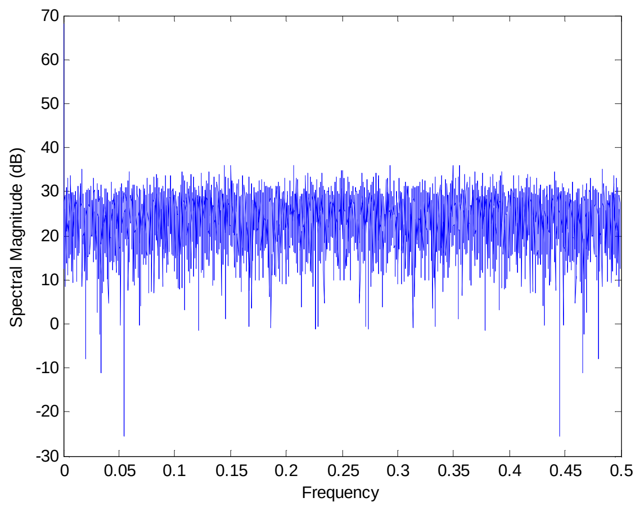

Figure 3: Fourier amplitudes for the system (1) with $x(0)=0.25$ and $(b,d)=(1.8,1.7)$. Sample size 5×10^3 points and initial 10^4 points discarded.

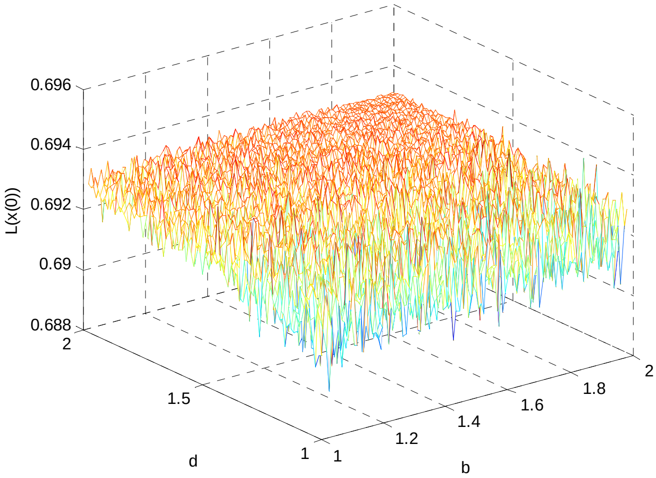

Figure 4: Lyapunov exponents of the systems (2). Initial value $x(0)=0.35$, sample size 5×10^3 points, and initial 10^4 points discarded.

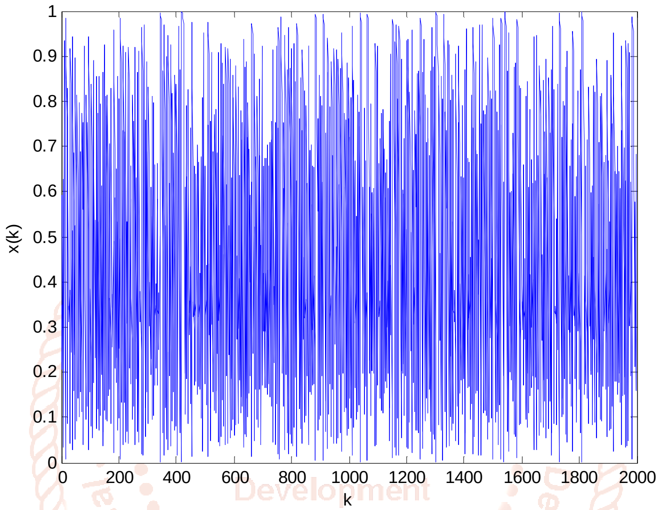

Figure 5: The time response of $x(k)$ for the system (2), with $x(0)=0.35$ and $(b,d)=(1.6,1.9)$.

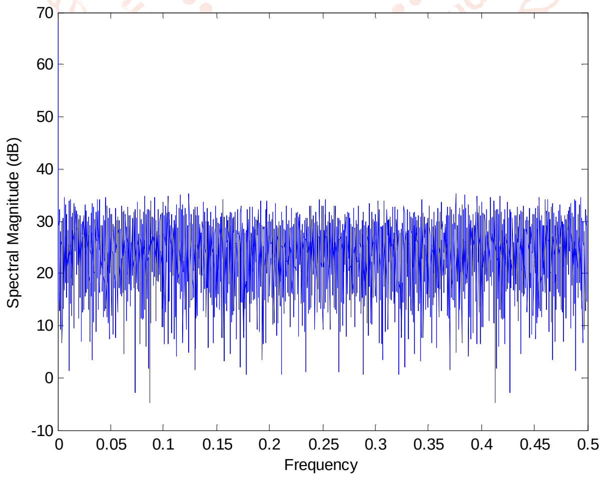

Figure 6: Fourier amplitudes for the system (2) with $x(0)=0.35$ and $(b,d)=(1.6,1.9)$. Sample size 5×10^3 points and initial 10^4 points discarded.