ABSTRACT
In this paper, two special standard cubic congruence of composite modulus of higher degree are considered for study to establish a formulation of the solutions of the congruence. The formulation of the solutions is established. The author’s rigorous labour works efficiently and succeed to formulate the solutions of the congruence under consideration. This paper presents the author’s successful efforts. The discovered formula is tested and verified true by illustrating different suitable examples. No effective method is found in the literature of mathematics for its solutions. Formulation is the merit of the paper. Formulation is proved time-saving, easy and simple. The study of standard cubic congruence of composite modulus of higher degree is now an interesting subject of mathematics.

KEYWORDS: Cubic Expansion, Cubic Congruence, Composite Modulus, Formulation

INTRODUCTION
A congruence of the type: \( x^3 \equiv a^3 \pmod{m} \), \( m \) being a composite positive integer is called a standard cubic congruence of composite modulus. Its solutions are the values of \( x \) that satisfies the congruence. Such types of congruence have different numbers of solutions. There are standard cubic congruence having unique solutions; some have three solutions; some have four solutions and some others have \( p \) or \( p^2 \) solutions; \( p \) being a positive prime integer.

LITERATURE REVIEW
The author has gone through many books of Number Theory but did not find any discussion or formulation on standard cubic congruence. Earlier mathematicians did not show any interest to study the congruence. Hence no formulation or any discussion is found.

The author intentionally started his research on standard cubic congruence and successfully discovered formulations of standard cubic congruence of prime and composite modulus \([1, 2, 3, 4, 5]\). The author already discovered a formulation of \( x^3 \equiv a^3 \pmod{m} \), \( m \geq 4 \). And \( x^3 \equiv 0 \pmod{m} \); \( m \leq 3 \) \([6]\).

Now he wishes to formulate the congruence of the type:
1. \( x^3 \equiv a^3 \pmod{m} \); \( m \geq 4 \); \( b \neq ka \); and also
2. \( x^3 \equiv 0 \pmod{m} \).

NEED OF RESEARCH
The author found no discussion or any formulation for the said congruence in the literature of mathematics. He tried his best to formulate the congruence. His efforts of establishment of the formulation is presented in this paper. This is the need of the research.

PROBLEM-STATEMENT
"To formulate a very special standard cubic congruence of composite modulus of the types
(1) \( x^3 \equiv a^3 \pmod{m} \); \( m \geq 4 \) & \( a, b \) any positive integers."
(2) \( x^3 \equiv 0 \pmod{m} \); \( m \leq 3 \) & \( a, b \) any positive integers."

ANALYSIS & DISCUSSION
Case-I: If \( x \equiv a^{m-2}, bk + a \), then
\[
x^3 \equiv \left( a^{m-2} \cdot bk + a \right)^3
\equiv \left( a^{m-2} \cdot bk \right)^3 + 3 \cdot \left( a^{m-2} \cdot bk \right)^2 \cdot a + \cdot 3 \cdot a^{m-2} \cdot bk \cdot a^2 + a^3 \pmod{m}.b \pmod{m}.
\equiv a^{m-2} \cdot bk \cdot (a^{m-2} \cdot bk + 3 \cdot a \cdot a^{m-2} \cdot bk + 3 \cdot a^2 \cdot a^3 \pmod{m}.b)
\equiv a^{m-2} \cdot bk \cdot (a^t) + a^3 \pmod{a^{m}.b),
\equiv a^m \cdot bk + a^3 \pmod{a^{m}.b},
\equiv a^3 \pmod{a^{m}.b}
\]

Thus, it can be said that \( x \equiv a^{m-2} \cdot bk + a \pmod{a^{m}.b} \) is a solution of it for some
k = 0, 1, 2, ... ... ... a^2 - 1, a^2, a^2 + 1, ........
It can also be seen that if k = a^2, a^2 + 1, ........, x ≡ a
m-2, a^2 + a = a^m + a
≡ a (mod a^m, b)
It is the same solution as for k = 0, 1, ........... , a^2 - 1.
Thus, the said congruence has a^2 incongruent solutions.

Case-II
Consider the congruence x^3 ≡ 0 (mod a^m, b).
It is seen that for x ≡ ab (Mod a^m, b); m ≤ 3,
x^3 ≡ (ab)^3 ≡ a^3b^3 ≡ 0 (mod a^3, b)
Thus, x ≡ ab (mod a^3, b) for t = 1, 2, 3, ...........
are the solutions of the congruence under consideration.

ILLUSTRATIONS

ILLUSTRATIONS OF CASE-I
Consider the congruence x^3 ≡ 3^3 (mod 5^4, 3).

It is of the type x^3 ≡ a^3 (mod a^m, b) with a = 3, m =
4, b = 3. It has 5^2 = 25 solutions.

These solutions are given by
x ≡ a^m-2, bk + a (mod a^m, b)
≡ 5^2-2, 3k + 5 (mod 5^2, 3)
≡ 5^2, 3k + 5 (mod 5^2, 3)
≡ 75k + 5 (mod 1875)
≡ 5, 80, 155, 230, 305, 380, 455, 530, 605, 680, 755, 830, 905,
980, 1055, 1130, 1205, 1280, 1355, 1430, 1505,
1580, 1655, 1730, 1805 (mod 1875).

These solutions are also verified true.
Consider another congruence x^3 ≡ 3^3 (mod 5^4, 16)
It is of the type x^3 ≡ a^3 (mod a^m, b) with a = 5, m =
4, b = 16. It has 5^2 = 25 solutions.

These solutions are given by
x ≡ a^m-2, bk + a (mod a^m, b)
≡ 5^4-2, 16k + 5 (mod 5^4, 16)
≡ 5^2, 16k + 5 (mod 5^2, 16)
≡ 400k + 5 (mod 10000)
≡ 5, 405, 805, 1205, 1605, 2005, 2405, 2805, 3205, 3605,
4005, 4405, 4805, 5205, 5605, 6005, 6405, 6805, 7205, 7605,
8005, 8405, 8805, 9205, 9605 (mod 10000).
These solutions are also verified true.

ILLUSTRATIONS of case-II
Consider the congruence x^3 ≡ 0 (mod 7^3, 4). It is of the type
x^3 ≡ 0 (mod a^m, b)
with m = 3, a = 7, b = 4.

Its solutions are given by
x ≡ ab (mod a^m, b) for any integer t.
i.e.x ≡ 7.4t = 28t (mod 7^3, 4) for t = 1, 2, 3, ...........
i.e.x ≡ 28, 56, 84, ........ .......... (mod 7^3, 4).

Consider the congruence x^3 ≡ 0 (mod 7^2, 4). It is of the type
x^3 ≡ 0 (mod a^m, b)
with m = 2, a = 7, b = 4.

Its solutions are given by
x ≡ ab (mod a^m, b) for any integer t.
i.e.x ≡ 7.4t = 28t (mod 7^2, 4) for t = 1, 2, 3, ...........
i.e.x ≡ 28, 56, 84, ........ .......... (mod 7^2, 4).
Consider the congruence x^3 ≡ 0 (mod 7^4). It is of the type
x^3 ≡ 0 (mod a^m, b)
with m = 1, a = 7, b = 4.

Its solutions are given by
x ≡ ab (mod a^m, b) for any integer t.
i.e.x ≡ 7.4t = 28t (mod 7^4) for t = 1, 2, 3, ...........
i.e.x ≡ 28, 56, 84, ........ .......... (mod 7^4).

Thus, it can be concluded that the congruence
x^3 ≡ a^3 (mod a^m, b); m ≥ 4, a, b positive integers has
a^2 incongruent solutions given by
x ≡ a^m-2, bk + a (mod a^m, b); k = 0, 1, 2, ...., a^2 - 1.
Similarly it can also be proved that
x = ab are the solutions of x^3 ≡ 0 (mod a^m, b) for
m = 1,2,3; t = 1, 2, 3, .............

REFERENCE


