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# Finding the Extreme Values with some Application of Derivatives

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**ABSTRACT**

There are many different way of mathematics rules. Among them, we express finding the extreme values for the optimization problems that changes in the particle life with the derivatives. The derivative is the exact rate at which one quantity changes with respect to another. And them, we can compute the profit and loss of a process that a company or a system. Variety of optimization problems are solved by using derivatives. There were use derivatives to find the extreme values of functions, to determine and analyze the shape of graphs and to find numerically where a function equals zero.

***KEYWORDS:*** *first order derivatives, second order derivatives, differentiation rules, related rate, optimization problems*

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## I. INTRODUCTION

In this paper, some basic definitions and notations of derivatives are firstly introduced. Next, some differentiation rules are presented. Moreover, related rates with some examples are also presented. Finally, some applications of derivatives are mentioned by using the closed interval method and the second derivative test.In calculus we have learnt that when y is the function of x, the derivative of y with respect to x (i.e $\frac{dy}{dx}$ ) measures rate of change in y with respect x. Geometrically, the derivative is the slope of curve at a point on the curve. The derivative is often called as the "instantaneous" rate of change. The process of finding the derivative is called as differentiation.

## II. FIRSR AND SECOND ORDER DERIVATIVES

If $y = f(x)$ is a differentiable function, then its derivative $f'(x)$ is also a function. If $f'$is also differentiable, then we can differentiate $f'$ to get a new function of x denoted by $f''$. So $f'' = (f')'$ .The function $f''$ is called the second derivative of $f$ because it is the derivative of the first derivative.

There are many ways to denote the derivative of a function $y = f(x)$, where the independent variable is x and the dependent variable is y. Some common alternative notations for the derivative are

$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{\mathrm{d}}{\mathrm{dx}} f(x) = D(f)(x) = D_x f(x)$. $f''(x) = y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy'}{dx} = \frac{df'(x)}{dx} = D^2(f)(x) = D_x^2 f(x)$.The symbols $\frac{d}{dx}$ and D indicate the operation of differentiation.

## III. SOME DIFFERENTIATION RULES

1. $\frac{d}{dx}(c) = 0$ ( c is constant )
2. $\frac{d}{dx} x^n = nx^{n-1}$ ( n is any real number )
3. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ ( u and v are differentiable at x)
4. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ( $v(x) \neq 0$)
5. If $y = f(u)$ *and* $u = g(x)$, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

## IV. RELATED RATE

Suppose we have two quantities, which are connected to each other and both changing with time. A related rates problem is a problem in which we know the rate of change of one of the quantities and want to find the rate of change of the other quantity.

Let the two variables be x and y. The relationship between them is expressed by a function $y = f(x)$. The rates of change in terms of their derivatives $\frac{dx}{dt}$ *and* $\frac{dy}{dt}$.If $\frac{dx}{dt}$ is known, we can determine $\frac{dy}{dt}$ (and vice versa).

## V. SOME EXAMPLES OF DERIVATIVES

### A. Example

Let $u = f(t)$ be the number of people in the labor force at time t in a given industry. (We treat this function as though it were differentiable even though it is an integer-valued step function.) Let $v = g(t)$ be the average production per person in the labor force at time t. The total production is then $y = uv$.

A. If the labor force is growing at the rate of 4% per year and the production per worker is growing at the rate of 5% per year, we can find the rate of growth of the total production, y.
B. If the labor force is decreasing at the rate of 2% per year while the production per person is increasing at the rate of 3% per year. Is the total production increasing, or is it decreasing, and at what rate?

$(a)$ $u = f(t)$ be the number of people in the labor force at time t

$v = g(t)$ be the average production per person in the labor force at time t.

The total production
$y = uv$

If the labor force is growing at the rate of 4% per year,
$\frac{du}{dt} = 0.04\, u$

If the production per worker is growing at the rate of 5% per year,
$\frac{dv}{dt} = 0.05v$

The rate of growth of the total production is
$\frac{dy}{dt} = \frac{d}{dt}(uv)$
$= u\frac{dv}{dt} + v\frac{du}{dt}$
$= u \times 0.05v + v \times 0.04u$
$= 0.05uv + 0.04uv$
$= 0.09uv = 0.09y$

The rate of growth of the total production is 9% per year.
(b)If the labor force is decreasing at the rate of 2% per year,
$\frac{du}{dt} = -0.02\, u$

If the production per person is increasing at the rate of 3% per year,
$\frac{dv}{dt} = 0.03v$

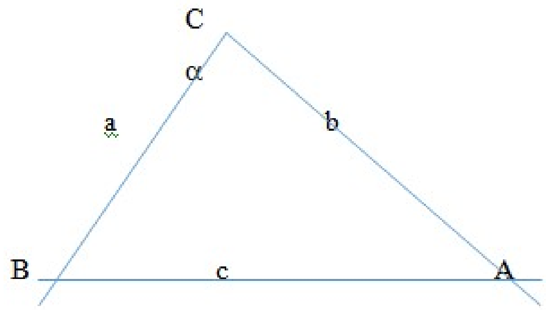
The rate of the total production is
$\frac{dy}{dt} = \frac{d}{dt}(uv)$
$= u\frac{dv}{dt} + v\frac{du}{dt}$
$= u \times 0.03v + v \times (-0.02u) = 0.01uv$
$= 0.01y.$

The total production is increasing at 1% per year.

## B. Example

A triangle has two sides a = 1 cm and b = 2cm. How fast is the third side c increasing when the angle α between the given sides is 60° and is increasing at the rate of 3° per second?

By given,
$a = 1cm, b = 2cm, \alpha = 60°, \frac{d\alpha}{dt} = 3°$




According to the law of cosines,
$c^2 = a^2 + b^2 - 2ab \cos\alpha$

We differentiate both sides of this equation with respect to time t,
$\frac{d}{dt}(c^2) = \frac{d}{dt}(a^2 + b^2 - 2ab \cos\alpha)$
$2c\frac{dc}{dt} = -2ab(-\sin\alpha)\frac{d\alpha}{dt}$ (a and b are constants)
$\frac{dc}{dt} = \frac{ab \sin\alpha}{c}\frac{d\alpha}{dt}$

Calculate the length of the side c
$c = \sqrt{a^2 + b^2 - 2ab\cos\alpha}$
$= \sqrt{1^2 + 2^2 - 2 \times 1 \times 2 \cos 60°}$
$= \sqrt{1 + 4 - 2}$
$= \sqrt{3}$ cm

Now we know all quantities to determine the rate of change $\frac{dc}{dt}$:
$\frac{dc}{dt} = \frac{ab \sin}{c}\frac{d\alpha}{dt}$
$= \frac{1 \times 2 \sin 60°}{\sqrt{3}} \times 3$
$= \frac{2\frac{\sqrt{3}}{2}}{\sqrt{3}} \times 3 = 3\, cm/sec$ .

# VI. APPLICATIONS OF DERIVATIVES

Derivatives have various applications in Mathematics, Science, and Engineering.

## A. Applied optimization

We use the derivatives to solve a variety of optimization problems in business, physics,
mathematics, and economics.

## B. Optimization Using the Closed Interval Method

The closed interval method is a way to solve a problem within a specific interval of a function. The solutions found by the closed interval method will be at the absolute maximum or minimum points on the interval, which can either be at the endpoints or at critical points.

## C. Critical point

We say that x = c is a critical point of the function $f(x)$ if $f(c)$ exist and if either of the
following are true.

$f'(c) = 0$ or $f'(c)$ doesn't exist.

If a point is not in the domain of the function then it is not a critical point.

## D. Example

A management company is going to build a new apartment complex. They know that if the complex contains x apartments the maintenance costs for the building, The land they have purchased can hold a complex of at most 500 apartments. How many apartments should the complex have in order to minimize the maintenance costs?

All we really need to do here is determine the absolute minimum of the maintenance function and the value of x that will give the absolute minimum.

$C(x) = 4000 + 14x - 0.04x^2$ ,
$0 \le x \le 500$

First, we'll need the derivative and the critical point that fall in the range $0 \le x \le 500$
$C'(x) = 14 - 0.08x$
$C'(x) = 0$ when $14 - 0.08x = 0$
$x = 175$

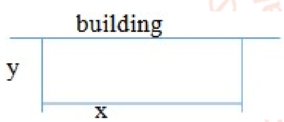
The critical point is $x = 175$
Since the cost function
$C(x) = 4000 + 14x - 0.04x^2$ ,
$0 \le x \le 500$

We can find the minimum value by the closed interval method:
$C(0) = 4000, C(175) = 5225, and$
$C(500) = 1000$

From these evaluations we can see that the complex should have 500 apartments to minimize the maintenance costs.

### E. Example

We need to enclose a rectangular field with a fence. We have 500 feet of fencing material and a building is on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area.




In this problem we want to maximize the area of a field and we know that will use 500 ft of fencing material. So, the area will be the function we are trying to optimize and the amount of fencing is the constraint. The two equations for these are,

Maximize: $A = xy$
Constraint: $500 = x + 2y$
$x = 500 - 2y$

Substituting this into the area function gives a function of y.
$A(y) = (500 - 2y)y$
$= 500y - 2y^2$

Now we want to find the largest value this will have on the interval [0,250]

The first derivative is
$A'(y) = 500 - 4y$
$A'(y) = 0$ when $500 - 4y = 0$
$y = 125$

We can find the maximum value by the closed interval method.

The largest possible area is $31250\ ft^2$.
$y = 125 \Rightarrow x = 500 - 2(125) = 250$

The dimensions of the field that will give the largest area, subject to the fact that we used exactly $500ft$ of fencing material, are 250 x 125.

## VII. SECOND DERIVATIVES TEST

Let I be the interval of all possible values of x in $f(x)$, the function we want to optimize,and suppose that f(x) is continuous on I, except possibly at the endpoints. Finally suppose that x=c is a critical point of $f(x)$ and that c is in the interval I. Then

1. If $f''(x) > 0$ for all x in I then $f(c)$ will be the absolute minimum value of $f(x)$ on the interval I.
2. If $f''(x) < 0$ for all x in I then $f(c)$ will be the absolute maximum value of $f(x)$ on the interval I.

(or)

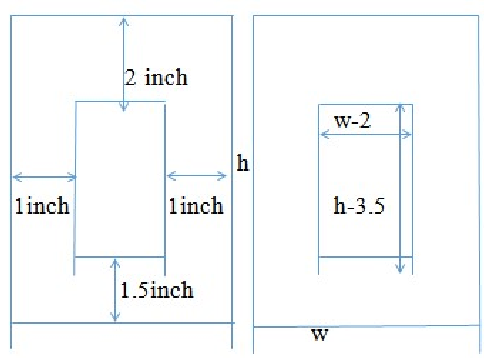
Suppose $f''$ is continuous on open interval that contains x = c.

1. If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at x = c.
2. $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at x = c.
3. $f'(c) = 0$ and $f''(c) = 0$, then the test fails. The function $f$ may have a local maximum, a local minimum, or neither. See [1].

## VIII. SOME EXAMPLES OF SECOND DERIVATIVE TEST

### A. Example

A printer need to make a poster that will have a total area of 200 in$^2$ and will have 1inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom. What dimensions will give the largest printed area?




The constraint is that the overall area of the poster must be 200 in$^2$ while we want to optimize the printed area. (i.e the area of the poster with the margins taken out ).

Let's define the height of the poster to be h and the width of the poster to be w. Here is a new sketch of the poster and we can see that once we've taken the margins into account the width of the printer area is w-2 and the height of the printer area is h-3.5.

Here are equations that we'll be working with
Maximize: $A = (w - 2)(h - 3.5)$

Solving the constraint for h and plugging into the equation for the printed area gives,

$$A\,(w) = (w-2)(\frac{200}{w} - 3.5)$$
$$= 200 - 3.5w - \frac{400}{w} + 7$$
$$= 207 - 3.5w - \frac{400}{w}$$

The first and second derivatives are

$$A'(w) = -3.5 + \frac{400}{w^2} = \frac{400-3.5w^2}{w^2}$$
$$A''(w) = -\frac{800}{w^3}$$

From the first derivative, we have the following two critical points (w = 0 is not critical point because the area function does not exit there).

$$w = \pm\sqrt{\frac{400}{3.5}} = \pm 10.6904$$

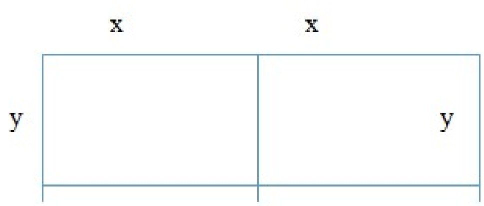
However, since we're dealing with the dimensions of a piece of paper we know that we must have w > 0 and so only 10.6904 will make sense. Also notice that provided w > 0 the second derivative will always be negative and so we know that the maximum printed area will be at w = 10.6904 inches.

The height of the paper that gives the maximum printed area is then

$$h = \frac{200}{10.6904} = 18.7084 \text{ inches.}$$

### B. Example

A 600 m$^2$ rectangular field is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?




In this problem we want minimize the total length of fence and we know that will use 600m$^2$ rectangular field.

Minimize: $P = 4x + 3y$ (x and y are the sides of the rectangle)

Constraint : $600 = 2xy$

Solving the constraint for y and plugging into the equation for the total length of fence gives,

$$P(x) = 4x + 3(\frac{600}{2x})$$
$$= 4x + \frac{900}{x}$$

The first and second derivatives are

$$P'(x) = 4 - \frac{900}{x^2} = \frac{4x^2-900}{x^2}$$
$$P''(x) = \frac{1800}{x^3}$$

From the first derivative, we have the following two critical points (x=0 is not a critical point because the function does not exist there).

$$x = \pm\sqrt{\frac{900}{4}} = \pm 15$$

However, since we're dealing with the dimensions of rectangle we know that we must have x > 0 and so only 15 will make sense.

Also notice that provided x > 0 the second derivative will always be positive and so we know that the minimum fence will be at x = 15 m.

The other side of rectangle is

$$y = \frac{600}{2\times 15} = 20 \text{ m}$$

The dimensions of the outer rectangle are
30 m by 20 m.
$P = 4\times 15 + 3\times 20 = 120$ m.
120 meters of fence will be needed.

## IX. CONCLUSION

We have use the derivatives to find the extreme values of a process in social life. In the study of the differential equations, we should know critical points because it is main point to find the extreme values. We compute increase or decrease function during a interval in our social environments. We use the derivative to determine the maximum and minimum values of particular function (e.g. cost, strength, amount of material used in a building, profit, loss, etc.)

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