International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 3 Issue 6, October 2019 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

Robust Exponential Stabilization for a Class of Uncertain Systems via a Single Input Control and its Circuit Implementation

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ABSTRACT

In this paper, the robust stabilization for a class of uncertain chaotic or non-chaotic systems with single input is investigated. Based on Lyapunov-like Theorem with differential and integral inequalities, a simple linear control is developed to realize the global exponential stabilization of such uncertain systems. In addition, the guaranteed exponential convergence rate can be correctly estimated. Finally, some numerical simulations with circuit realization are provided to show the effectiveness of the obtained result.

KEYWORDS: *Uncertain systems, robust exponential stabilization, Zhu chaotic system, single input control*

How to cite this paper: Yeong-Jeu Sun "Robust Exponential Stabilization for a Class of Uncertain Systems via a Single Input Control and its Circuit Implementation" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-3 | Issue-6, October 2019, pp.1202-1205, URL: https://www.ijtsrd.com/papers/ijtsrd29322.pdf

IJTSRD29322

1. INTRODUCTION

In the past, various chaotic systems have been discussed, studied, and applied in physical systems; such as nonlinear systems and uncertain systems; see, for instance, [1]-[16] and the references therein. The scientific community has confirmed that chaos is not only one of the factors that cause system instability, but also the source of system oscillations. For chaotic systems, designing controllers to remove unstable or oscillating factors is often the mission of most researchers and engineers.

On the other hand, the linear controller is not only easy to implement, but also has the advantage of being inexpensive. For an uncertain chaotic control system, designing a practical and economical linear controller is an important issue today. The number of controllers often affects the price of the overall controller. Therefore, how to reduce the number of controllers is a big challenge for experts and scholars. Generally speaking, due to the uncertainty of the system parameters and the error of the system modeling, the real system can be presented in the form of an uncertain system to reflect the truth of the system. Therefore, in order to fully present the real physical system, most scholars analyze or design for uncertain systems.

Based on the above mentioned reasons, this paper aims at a class of uncertain chaotic control systems, using Lyapunov-like Theorem with differential and integral inequalities to design a single and linear controller to ensure that the closed-loop control system achieves global exponential stability. Throughout this paper, $\Re^n$ denotes the n-dimensional Euclidean space, $\|x\|$ denotes the Euclidean norm of the vector $x \in \Re^n$, $|a|$ denotes the modulus of a real number a, and $\lambda_{\min}(P)$ denotes the minimum eigenvalue of the matrix P with real eigenvalues.

2. PROBLEM FORMULATION AND MAIN RESULTS

Consider the following uncertain nonlinear systems with single input described by

$$\dot{x}_1(t) = \Delta a \cdot x_1(t) + \Delta d_1 \cdot x_2(t) + f_1(x_1, x_2, x_3), \tag{1a}$$

$$\dot{x}_2(t) = \Delta d_2 \cdot x_1(t) + \Delta d_3 \cdot x_2(t) + f_2(x_1, x_2, x_3) + \Delta b \cdot u(t), \tag{1b}$$

$$\dot{x}_3(t) = \Delta c \cdot x_3(t) + f_3(x_1, x_2, x_3), \tag{1c}$$

where $x(t) := [x_1(t) \quad x_2(t) \quad x_3(t)]^T \in \Re^{3\times 1}$ is the state vector, $u(t) \in \Re$ is the control input, $\Delta a, \Delta b, \Delta c,$ and Δd_i are uncertain parameters, and f_i is nonlinear term with $f_i(0,0,0) = 0, \forall i \in \{1,2,3\}$. In addition, for the solution existence of the uncertain nonlinear systems (1), we assume that f_1, f_2, and f_3 are smooth functions.

Throughout this paper, the following assumptions are made:

(A1) There exist constants $\overline{a}, \underline{a}, \overline{b}, \underline{b}, \overline{c}, \underline{c}$, and $\overline{d_i}$ such that

$$-\overline{a} \le \Delta a \le -\underline{a} < 0, \quad 0 < \underline{b} \le \Delta b \le \overline{b},$$
$$-\overline{c} \le \Delta c \le -\underline{c} < 0,$$
$$|\Delta d_i| \le \overline{d_i}, \quad \forall\, i \in \{1,2,3\}.$$

(A2) There exist positive numbers k_1, k_2, and k_3 such that

$$\sum_{i=1}^{3} k_i \cdot x_i \cdot f_i(x_1, x_2, x_3) = 0.$$

Remark 1. In fact, Zhu chaotic system [16] is a special case of the uncertain systems (1) with

$$\Delta a = -1, \quad \Delta d_1 = 1.5, \quad f_1(x_1,x_2,x_3) = x_2 x_3,$$
$$\Delta d_2 = 0, \quad \Delta d_3 = 2.5, \quad f_2(x_1,x_2,x_3) = -x_1 x_3,$$
$$\Delta b = 0, \quad \Delta c = -4.9, \quad f_3(x_1,x_2,x_3) = x_1 x_2.$$

The global exponential stabilization and the exponential convergence rate of the system (1) are defined as follows.

Definition 1. The system (1) is said to be globally exponentially stable if there exist a control u and positive numbers α and k, such that

$$|x_i(t)| \le k \cdot e^{-\alpha t}, \quad \forall t \ge 0, \; i \in \{1,2,3\}.$$

In this case, the positive number α is called the exponential convergence rate.

The objective of this paper is to search a simple linear control such that the global exponential stabilization of uncertain nonlinear system (1) can be guaranteed. Furthermore, an estimate of the exponential convergence rate of such stable systems is also studied.

Now we present the main result for the global exponential stabilization of uncertain nonlinear systems (1) with (A1) and (A2).

Theorem 1. The uncertain nonlinear system (1) with (A1) and (A2) is globally exponentially stabilizable at the zero equilibrium point by the linear control

$$u = -r x_2, \tag{2}$$

with

$$r > \frac{\overline{d_3}}{\underline{b}} + \frac{\left(k_1 \overline{d_1} + k_2 \overline{d_2}\right)^2}{4 k_1 k_2 \underline{a}\underline{b}}. \tag{3}$$

Besides, the guaranteed exponential convergence rate is given by

$$\frac{\lambda_{\min}(P)}{2 \max\{k_1, k_2, k_3\}}, \tag{4}$$

with

$$P := \begin{bmatrix} 2k_1 \underline{a} & -\left(k_1 \overline{d_1} + k_2 \overline{d_2}\right) & 0 \\ -\left(k_1 \overline{d_1} + k_2 \overline{d_2}\right) & 2k_2\left(\underline{b} r - \overline{d_3}\right) & 0 \\ 0 & 0 & 2k_3 \underline{c} \end{bmatrix}.$$

Proof. It can be readily obtained that $\det([2k_1 \underline{a}]) > 0$, $\det\left(\begin{bmatrix} 2k_1 \underline{a} & -\left(k_1 \overline{d_1} + k_2 \overline{d_2}\right) \\ -\left(k_1 \overline{d_1} + k_2 \overline{d_2}\right) & 2k_2\left(\underline{b} r - \overline{d_3}\right) \end{bmatrix}\right) > 0$, and $\det(P) > 0$, in view of (A1), (A2), and (3). This implies that the matrix of P is positive definite. Let

$$V(x(t)) := k_1 \cdot x_1^2(t) + k_2 \cdot x_2^2(t) + k_3 \cdot x_3^2(t). \tag{5}$$

Obviously, one has

$$\begin{aligned} &\min\{k_1, k_2, k_3\} \cdot \|x(t)\|^2 \\ &\le V(x(t)) \\ &\le \max\{k_1, k_2, k_3\} \cdot \|x(t)\|^2. \end{aligned} \tag{6}$$

The time derivative of $V(x(t))$ along the trajectories of dynamical error system, with (1)-(6) and (A1)-(A2), is given by

$$\begin{aligned} \dot{V}(x(t)) &= 2k_1 \cdot x_1 \dot{x}_1 + 2k_2 \cdot x_2 \dot{x}_2 + 2k_3 \cdot x_3 \dot{x}_3 \\ &= 2k_1 x_1 (\Delta a x_1 + \Delta d_1 x_2 + f_1) \\ &\quad + 2k_2 x_2 (\Delta d_2 x_1 + \Delta d_3 x_2 + f_2 + \Delta b \cdot u) \\ &\quad + 2k_3 x_3 (\Delta c x_3 + f_3) \\ &\le -2k_1 \underline{a} x_1^2 + 2k_1 \overline{d_1} |x_1||x_2| \\ &\quad + 2k_2 \overline{d_2} |x_1||x_2| + 2k_2 \overline{d_3} x_2^2 \\ &\quad - 2k_3 \underline{c} x_3^2 \\ &\quad + 2(k_1 x_1 f_1 + k_2 x_2 f_2 + k_3 x_3 f_3) \\ &\quad - 2k_2 \Delta b r x_2^2 \\ &\le -2k_1 \underline{a} x_1^2 + 2\left(k_1 \overline{d_1} + k_2 \overline{d_2}\right)|x_1||x_2| \\ &\quad + 2k_2 \overline{d_3} x_2^2 - 2k_3 \underline{c} x_3^2 - 2k_2 \underline{b} r x_2^2 \\ &= -\left[|x_1| \;\; |x_2| \;\; |x_3|\right] P \left[|x_1| \;\; |x_2| \;\; |x_3|\right]^T \\ &\le -\lambda_{\min}(P) \|x(t)\|^2, \quad \forall\, t \ge 0. \\ &\le -\frac{\lambda_{\min}(P)}{\max\{k_1, k_2, k_3\}} V(t), \quad \forall\, t \ge 0. \end{aligned}$$

Thus, one has

$$\begin{aligned} &e^{\frac{\lambda_{\min}(P)}{\max\{k_1,k_2,k_3\}} t} \cdot \dot{V} \\ &+ e^{\frac{\lambda_{\min}(P)}{\max\{k_1,k_2,k_3\}} t} \cdot \left[\frac{\lambda_{\min}(P)}{\max\{k_1,k_2,k_3\}}\right] \cdot V \\ &= \frac{d}{dt}\left[e^{\frac{\lambda_{\min}(P)}{\max\{k_1,k_2,k_3\}} t} \cdot V\right] \\ &\le 0, \quad \forall\, t \ge 0. \end{aligned}$$

It follows that

$$\begin{aligned} &\int_0^t \frac{d}{dt}\left[e^{\frac{\lambda_{\min}(P)}{\max\{k_1,k_2,k_3\}} t} \cdot V(x(t))\right] dt \\ &= e^{\frac{\lambda_{\min}(P)}{\max\{k_1,k_2,k_3\}} t} \cdot V(x(t)) - V(x(0)) \\ &\le \int_0^t 0 \, d\tau = 0, \quad \forall\, t \ge 0. \end{aligned} \tag{7}$$

From (6) and (7), it is easy to obtain that

$$\begin{aligned}&\min\{k_1,k_2,k_3\}\cdot\|x(t)\|^2\\ &\le V(x(t))\\ &\le e^{\frac{-\lambda_{\min}(P)}{\max\{k_1,k_2,k_3\}}t}V(x(0))\\ &\le e^{\frac{-\lambda_{\min}(P)}{\max\{k_1,k_2,k_3\}}t}\cdot\max\{k_1,k_2,k_3\}\cdot\|x(0)\|^2,\ \forall t\ge 0.\end{aligned}$$

Consequently, we conclude that

$$\begin{aligned}&\|x(t)\|\\ &\le\sqrt{\frac{\max\{k_1,k_2,k_3\}}{\min\{k_1,k_2,k_3\}}}\cdot\|x(0)\|\cdot e^{\frac{-\lambda_{\min}(P)}{2\max\{k_1,k_2,k_3\}}t},\\ &\forall t\ge 0.\end{aligned}$$

This completes the proof. □

3. NUMERICAL SIMULATIONS WITH CIRCUIT IMPLEMENTATION

Example 1. Consider the system (1) with

$$-1.2\le\Delta a\le -0.8,\quad 1\le\Delta b\le 1.2, \tag{8a}$$

$$-5.2\le\Delta c\le -4.7,\ |\Delta d_1|\le 1.6,\ |\Delta d_2|\le 0.2, \tag{8b}$$

$$|\Delta d_3|\le 2.7,\quad f_1(x_1,x_2,x_3)=x_2x_3, \tag{8c}$$

$$f_2(x_1,x_2,x_3)=-x_1x_3,\ f_3(x_1,x_2,x_3)=x_1x_2. \tag{8d}$$

By selecting the parameters $\overline{a}=1.2$, $\underline{a}=0.8$, $\overline{b}=1.2$, $\underline{b}=1$, $\overline{c}=5.2$, $\underline{c}=4.7$, $\overline{d_1}=1.6$, $\overline{d_2}=0.2$, $\overline{d_3}=2.7$, and $k_1=k_3=1$, $k_2=2$, (A1)-(A2) are evidently satisfied.

Thus, one has $\dfrac{\overline{d_3}}{\underline{b}}+\dfrac{\left(k_1\overline{d_1}+k_2\overline{d_2}\right)^2}{4k_1k_2\underline{a}\underline{b}}=3.325$.

Consequently, by Theorem 1 with the choice $r=4$, we conclude that the system (1) with (8) and $u=-4x_2$, is globally exponentially stable. In this case, from (4), the guaranteed exponential convergence rate is given by

$$\frac{\lambda_{\min}(P)}{2\max\{k_1,k_2,k_3\}}=0.18.$$

The typical state trajectories of the uncontrolled system and the feedback-controlled system are depicted in Fig. 1 and Fig. 2, respectively. Besides, the control signal and the electronic circuit to realize such a control law are depicted in Fig. 3 and Fig. 4, respectively.

4. CONCLUSION

In this paper, the robust stabilization for a class of uncertain nonlinear systems with single input has been investigated. Based on Lyapunov-like Theorem with differential and integral inequalities, a simple linear control has been developed to realize the global exponential stabilization of such uncertain systems. In addition, the guaranteed exponential convergence rate can be correctly estimated. Finally, some numerical simulations with circuit realization have been provided to show the effectiveness of the obtained result.

ACKNOWLEDGEMENT

The author thanks the Ministry of Science and Technology of Republic of China for supporting this work under grant MOST 107-2221-E-214-030. Furthermore, the author is grateful to Chair Professor Jer-Guang Hsieh for the useful comments.

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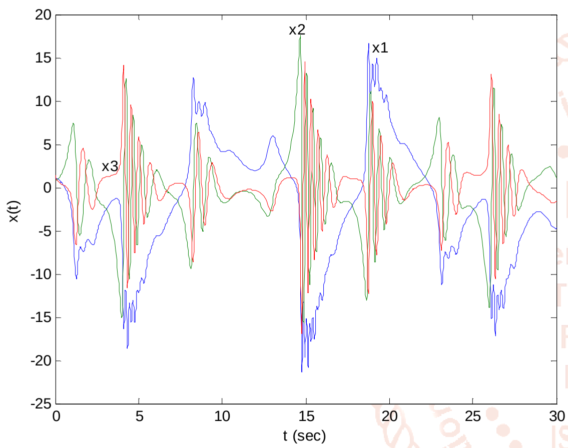

Figure 1: Typical state trajectories of the uncontrolled system of Example 1.

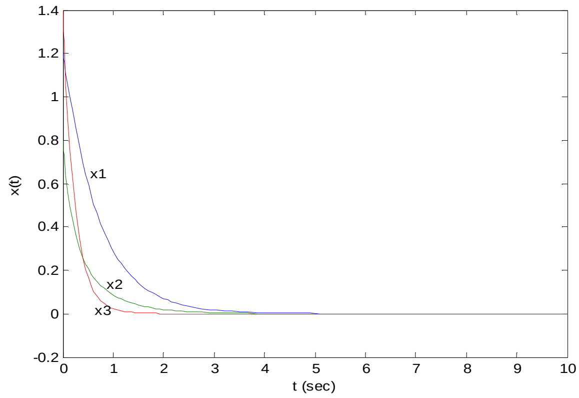

Figure 2: Typical state trajectories of the feedback-controlled system of Example 1.

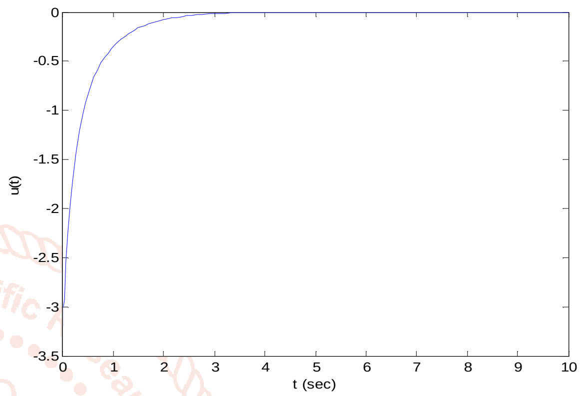

Figure 3: Control signal of Example 1.

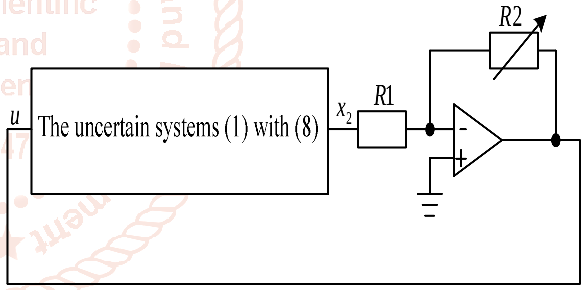

Figure 4: The diagram of implementation of Example 1, where $R1 = 10k\Omega$ **and** $R2 = 40k\Omega$.