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Characteristics of Fuzzy Wheel Graph and Hamilton Graph with Fuzzy Rule

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ABSTRACT
Graph theory is the concepts used to study and model various application in different areas. We proposed the wheel graph with n vertices can be defined as 1-skeleton of on (n-1) gonal pyramid it is denoted by $w_n$with n+ 1 vertex (n≥3). A wheel graph is hamiltonion, self-dual and planar. In the mathematical field of graph theory, and a **Hamilton path**(or **traceable graph**) is a path in an undirected or directed graph that visits each vertex exactly once. A Hamiltonian cycle is a hamiltonian path that is a cycle. In this paper, we consider the wheel graph and also the hamilton graph using **if-then-rules** fuzzy numbers. The results are related to the find the degree of odd vertices and even vertices are same by applying **if-then-rules** through the paths described by fuzzy numbers.

KEYWORDS: *Degree of Vetex, Incident Graph, Fuzzy Wheel Graph, Hamilton graph, Fuzzy IF-THEN rule*

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I. INTRODUCTION

The first definition of fuzzy graph was introduced by Kaufmann (1973), based on Zadeh's fuzzy relations (1971) A more elaborate definition is due to Azriel Rosenfeld who considered fuzzy relation on fuzzy sets and developed the theory of fuzzy graph in 1975.

During the same time Yeh and Bang have also introduced various connectedness concepts in fuzzy graph. Till now fuzzy graphs has been witnessing a tremendous growth and finds applications in many branches of engineering and technology.

Fuzzy systems based on fuzzy if-then rules have been successfully applied to various theorems in the field of fuzzy control. Fuzzy Rule based system has high comprehensibility because human users can easily understand the meaning of each fuzzy if-then rule through its linguistic interpretation.

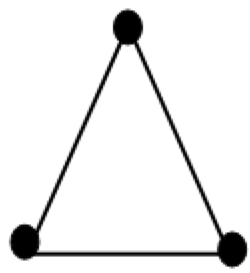
Graph theory has numerous applications to problems in system analysis, operation research, transportation and economics. In many cases, however some aspects of graph theoretic problem may be uncertain. For example, the vehicle travel time or vehicle capacity on a road network may not be known exactly. In such cases, it is natural to deal with the uncertainly using fuzzy set theory. The concepts of a fuzzy graph are a natural generalization of crisp graphs, given by Rosenfeld [Zadeh et al., 1975].

A wheel graph is a graph formed by connecting a single universal vertex to all vertices of a cycle. A wheel graph with n-vertices can also be defined as the 1-skeloton of an (n-1)gonal pyramid.

In the mathematical field of graph theory ,a **Hamilton path**(or **traceable graph**) is a path in an undirected or directed graph that visits each vertex exactly once. A Hamiltonian cycle is a hamiltonian path that is a cycle.

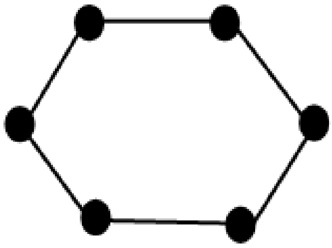
II. BASIC CONCEPT

Definition 2.1:
A graph G consists of a pair G: (V,E) where V(G) is a non empty finite set whose elements are called points or vertices and E(G) is a set of unordered pairs of distinct elements of V(G). The elements of E(G) are called lines or edges of the graph G.



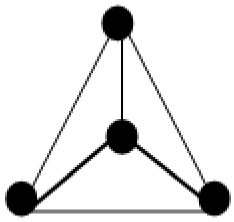
Definition 2.2: Cycle graph

In graph theory, a cycle graph or circular graph is a graph that consists of a single cycle, or in other words, some number of vertices (at least 3)connected in a closed chain. The cycle graph with n vertices is calledc_n. The number of vertices in$c_n$equals the number of edges, and every vertex has degree 2; that is, every vertex has exactly two edges incident with it.



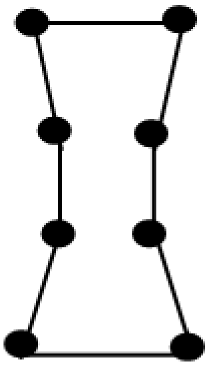
Definition 2.3:

A wheel graph is obtained from a cycle graph C_{n-1} by adding a new vertex. That new vertex is called a **Hub** which is connected to all the vertices of C_n.



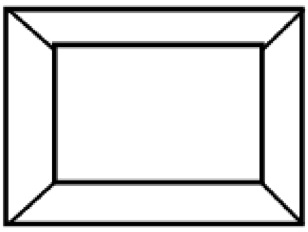
Definition 2.4: Hamilton cycle

A path that contains every vertex of G is called a Hamilton path of G. similarly a Hamilton cycle of G is a cycle that contains every vertex of G



Definition 2.5: Hamilton graph

A graph Hamiltonian if it contains a Hamilton cycle is called Hamilton graph.



Definition 2.6:

Let V be a non- empty set. A fuzzy graph is a pair of function G :(σ,μ), where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ. i.e.,σ:V→ [0,1] such that μ(u,v)≤ $\sigma(u)$^$\sigma(v)$ for all u, v in V. where , uv denotes the edge between u and v and $\sigma(u)$^$\sigma(v)$ denotes the minimum of $\sigma(u) and\ \sigma(v)$.

Definition 2.7: Incident

When a vertex $\sigma(u_i)$ is an end vertex of some edges $\mu(u_{i,}v_j)$ of any fuzzy graph G:(σ,μ).Then $\sigma(u_i)$ and $\mu(u_{i,}v_j)$ are said to be incident to each other.

Definition 2.8:

The degree of anyvertex$\sigma(u_i)$ of a fuzzy graph is sum of degree of membership of all those edges which are incident on vertex $\sigma(u_i)$ and is denoted by d[$\sigma(u_i)$].

Definition 2.9: Fuzzy If-Then Rule:

A fuzzy rule is defined as a conditional Statement in the form:

IF x is A, THEN y is B; where x and y are linguistic variable; A and B are linguistic values determined by fuzzy sets on the universe of discourse XandY, respectively.

- A rule is also called a fuzzy implication.
- "x is A" is called the antecedent or premise.
- "y is B" is called the consequenceor conclusion.

Example:

- IF pressure is high, THEN volume is small.
- IF the speed is high, THEN apply the brake a little.

III. FUZZY WHEEL GRAPH WITH FUZZY RULE

Theorem:

In fuzzy wheel graph G, the sum of degrees of vertices of even degree is equal to twice the degree of membership of all the edges and the difference of the sum of degrees of vertices of odd degree.

Proof

Let G:(σ,μ) is fuzzy wheel graph. Consider 11-vertices {$\sigma(v_1),\sigma(v_2),\sigma(v_3),\sigma(4),.\sigma(v_{11})$} of fuzzy wheel graph G:(σ,μ)

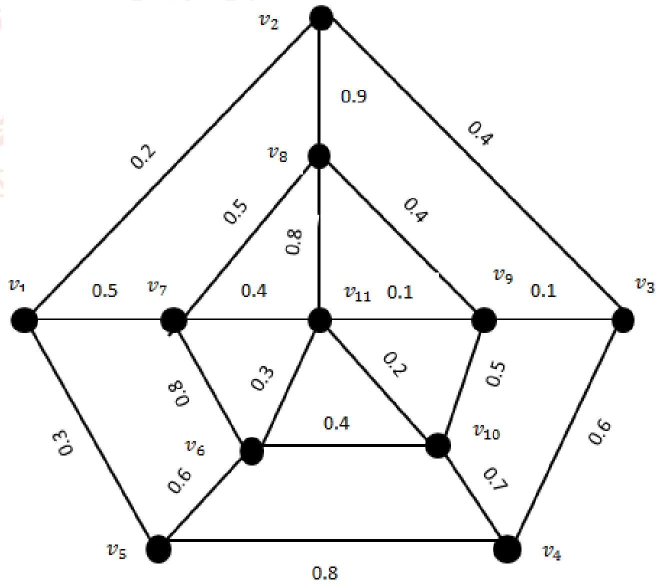


Figure: Fuzzy wheel graph

IF the membership grades of edges which are incident on any degree of vertexσ( ) are added, THEN the sum of corresponding membership value of vary.

$d[\sigma(v_1)] = 0.2 + 0.3 + 0.5 = 1$

$d[\sigma(v_2)] = 0.2 + 0.9 + 0.4 = 1.5$

$d[\sigma(v_3)] = 0.1 + 0.4 + 0.6 = 1.1$

$d[\sigma(v_4)] = 0.6 + 0.7 + 0.8 = 2.1$

$d[\sigma(v_5)] = 0.8 + 0.6 + 0.3 = 1.7$

$d[\sigma(v_6)] = 0.8 + 0.3 + 0.4 + 0.6 = 2.1$

$d[\sigma(v_7)] = 0.4 + 0.5 + 0.8 + 0.5 = 2.2$

$d[\sigma(v_8)] = 0.5 + 0.8 + 0.4 + 0.9 = 2.6$

$d[\sigma(v_9)] = 0.1 + 0.5 + 0.4 + 0.1 = 1.1$

$d[\sigma(v_{10})] = 0.5 + 0.2 + 0.4 + 0.7 = 1.8$

$d[\sigma(v_{11})] = 0.3 + 0.4 + 0.8 + 0.1 + 0.2 = 1.8$

IF the membership grades of edges are added, THEN we find the degree of edges,

$\sum_{i=1}^{11} \mu(u_i, v_{i+1})$=0.4+0.6+0.8+0.3+0.2+0.9+0.1+0.7+0.6+0.5+0.5+0.4+0.5+0.4+0.8+0.4+0.8+0.1+0.2+0.3=9.5

$\sum_{i=1}^{11} \mu(u_i, v_{i+1})$=9.5

$\sum_{i=1}^{11} d[\sigma(v_i)]$=Twice the sum of degree of membership of (u_i, v_{i+1})

Therefore,$\sum_{i=1}^{n} d(v_i)=2\sum_{i=1}^{n} \mu(u_i, v_{i+1})$

But here,
The deg$[\sigma(v_i)]$ has been splitied into two parts.

i.e.,$\sum_{i=1}^{k} deg(v_i)+\sum_{i=1}^{n} deg(w_k)=2\sum_{i=1}^{n} \mu(u_i, v_{i+1})$

$\sum_{i=1}^{k} d(v_i)$ denotes the sums over even degree vertices, i.e.,$\sigma(v_2), \sigma(v_4), \sigma(v_6), \sigma(v_8), \sigma(v_{10})$.

$\sum_{i=1}^{k} d(v_i)$=1.5+2.1+2.1+2.6+1.8

$\sum_{i=1}^{k} d(v_i)$=10.1

Now,$\sum_{i=1}^{n} d(w_k)$ denotes the sum over odd degree vertices, i.e., $\sigma(v_1), \sigma(v_3), \sigma(v_5), \sigma(v_7), \sigma(v_9), \sigma(v_{11})$

$\sum_{i=1}^{n} d(w_k)$=1+1.1+1.7+2.2+1.8+1.1

$\sum_{i=1}^{n} d(w_k)$=8.9

Hence If the sum of degrees of vertices of even degree is10.1, then it is equal to twice the degree of membership of all edges and difference of the sum of degrees of vertices of odd degree.

$\sum_{i=1}^{k} d(v_i)$= $2\sum_{i=1}^{n} \mu(u_i, v_{i+1})$-$\sum_{i=1}^{n} d(w_k)$

10.1=2(9.5)-8.9

=19-8.9

10.1=10.1

Hence the theorem

Theorem:
In fuzzy Hamilton graph G, the sum of degrees of vertices of even degree is equal to twice the degree of membership of all the edges and the difference of the sum of degrees of vertices of odd degree.

Proof:
Let G:(σ,μ) is fuzzy Hamilton graph. Consider 6-vertices $\{\sigma(v_1), \sigma(v_2), \sigma(v_3), \sigma(v_4), \sigma(v_5), \sigma(v_6)\}$ of fuzzy wheel graph G:(σ,μ)

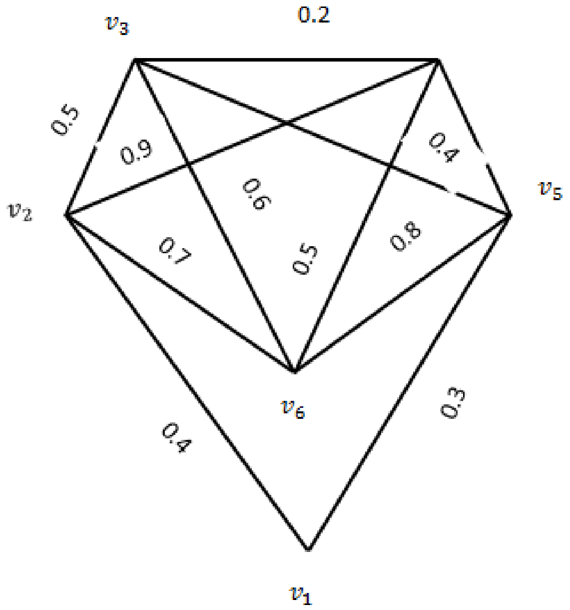


Figure: Fuzzy Hamilton graph

IF the membership grades of edges which are incident on any degree of vertex σ( ) are added, THEN the sum of corresponding membership value of vary.

$d[\sigma(v_1)] = 0.4 + 0.3 = 0.7$

$d[\sigma(v_2)] = 0.4 + 0.7 + 0.9 + 0.5 = 2.5$

$d[\sigma(v_3)] = 0.5 + 0,2 + 0.6 + 0.4 = 1.7$

$d[\sigma(v_4)] = 0.2 + 0.5 + 0.5 + 0.9 = 2.1$

$d[\sigma(v_5)] = 0.5 + 0.3 + 0.4 + 0.8 =2$

$d[\sigma(v_6)] = 0.7 + 0.6 + 0.5 + 0.8 = 2.6$

IF the membership grades of edges are added, THEN we find the degree of edges,

$\sum_{i=1}^{6} \mu(u_i, v_{i+1})$=0.4+0.5+0.2+0.5+0.3+0.7+0.9+0.6+0.5+0.8+0.4=5.8

$\sum_{i=1}^{6} \mu(u_i, v_{i+1})$=5.8

$\sum_{i=1}^{6} d[\sigma(v_i)]$ =Twice the sum of degree of membership of (u_i, v_{i+1})

Therefore,$\sum_{i=1}^{n} d(v_i)=2\sum_{i=1}^{n} \mu(u_i, v_{i+1})$

But here,

The deg$[\sigma(v_i)]$ has been splitied into two parts.

i.e.,$\sum_{i=1}^{k} deg(v_i)+\sum_{i=1}^{n} deg(w_k)=2\sum_{i=1}^{n} \mu(u_i, v_{i+1})$

$\sum_{i=1}^{k} d(v_i)$ denotes the sums over even degree vertices, i.e.,$\sigma(v_2), \sigma(v_4), \sigma(v_6)$,

$\sum_{i=1}^{k} d(v_i)$=2.5+2.1+2.6

$\sum_{i=1}^{k} d(v_i)$=7.2

Now,$\sum_{i=1}^{n} d(w_k)$ denotes the sum over odd degree vertices, i.e., $\sigma(v_1), \sigma(v_3), \sigma(v_5)$,

$\sum_{i=1}^{n} d(w_k)$=0.7+1.7+2

$\sum_{i=1}^{n} d(w_k)$=4.4

If the sum of degrees of vertices of even degree is7.2, it is equal to twice the degree of membership of all edges and difference of the sum of degrees of vertices of odd degree.

$\sum_{i=1}^{k} d(v_i)= 2\sum_{i=1}^{n} \mu(u_i\, , v_{i+1})$-$\sum_{i=1}^{n} d(w_k)$

7.2=2(5.8)-4.4

7.2=7.2

Hence the theorem

IV. CONCLUSION

In this paper we obtain that wheel graph wn, and hamilton path. We conclude that the odd degree of vertices is equal to the even degree of vertices, for both the above cases with the help of IF-THEN rules (i.e., L.H.S=R.H.S).Finally, IF-THEN rules applied through shortest paths are shown with different analysis.

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