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Comparison of Several Numerical Algorithms with the use of Predictor and Corrector for solving ode

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ABSTRACT
In this paper, we introduce various numerical methods for the solutions of ordinary differential equations and its application. We consider the Taylor series, Runge-Kutta, Euler's methods problem to solve the Adam's Predictor, Corrector and Milne's Predictor, Corrector to get the exact solution and the approximate solution.

KEYWORDS: *Ordinary differential equation, Initial Condition, Adams Predictor, Corrector and Milne's Predictor, Corrector*

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I. INTRODUCTION

Numerical methods for ordinary differential equations are methods used to find numerical approximations to the solutions of ordinary differential equation .Their use is also known as "numerical integration", although this term is sometimes taken to mean the computation of integrals.

Many differential equation cannot be solved using symbolic computation. For practical purposes, however-such as in engineering-a numeric approximation to the solution is often sufficient. The algorithms studied here can be used to compute such an approximation. An alternative method is to use techniques from calculus to obtain a series expansion of the solution.

Ordinary differential equations occur in many scientific disciplines, for instance in physics, chemistry, biology, and economics. In addition, some methods in numerical partial differential equation convert the partial differential equation into an ordinary differential equation, which must then be solved.

A general form of the type of problem we consider is

$$\begin{cases} \frac{dy}{dx} = f(x, y),\ x \in [a, b] \\ Y(a) = y_0 \end{cases}$$

First the interval [a, b] is divided into an equal parts, x_i = a +ih, (i=0, 1, 2 …n), the step is h=x_{i+1}-x_i.

Then to solve the function y(x) in a series of discrete equidistant node $x_0<x_1<x_3<\ldots\ldots<x_n$ to get approximate values $y_0<y_1<y_2<y_3<\ldots\ldots<y_n$.

II. PREDICTOR-CORRECTOR METHODS:

The methods which we have discussed so far are called single-step methods because they use only the information from the last step computed. The methods of Milne's predictor-corrector, Adams-Bash forth Predictor corrector formulae are multi-step methods.

In solving the equation $\frac{dy}{dx}$ =f(x, y), y (x_0) =y_0 we used Euler's formula

$$y_{i+1}=y_i+h\, f'(x_i,y_i),\ i=0, 1, 2 \qquad \ldots\ldots\ldots\ldots (1)$$

We improved this value by Improved Euler method

$$y_{i+1}=y_i+\frac{1}{2} h\,[f(x_i,y_i) + f(x_{i+1},y_{i+1})] \qquad \ldots\ldots\ldots\ldots (2)$$

In the equation (2), to get the value of y_{i+1} we require y_{i+1} on the R.H.S. To overcome this difficulty, we us we predict a value of y_{i+1} from the rough formula (1) and use in (2) to correct the value. Every time, we improve using (2). Hence equation (1) Euler's formula is a predictor and (2) is a corrector. A predictor formula is used to predict the value of y at x_{i+1} and a corrector formula is used to corrector the error and to improve that value ofy_{i+1}.

2.1. Milne's Predictor Corrector Formula

The Milne-Simpson method is a Predictor method. It uses a Milne formula as a Predictor and the popular Simpson's formula as a Corrector. These formulae are based on the fundamental theorem of calculus.

$$Y(x_{i+1}) = y(x_j) + \int_{x_j}^{x_{i+1}} f(x,y)dx \quad \text{............ (3)}$$

When j=i-3, the equation .becomes an open integration formula and produces the Milne's formulae

$$y_{i+1,p}=y_{i-3}+\frac{4h}{3}(2f_{i-2}-f_{i-1}+2f_i) \quad \text{............ (4)}$$

Similarly, When j=i-1, equation becomes a closed form integration and produces the two-segment Simpson's formula

$$y_{i+1,c}=y_{i+1}+\frac{h}{3}(f_{i-1}+4f_i+f_{i+1}) \quad \text{............ (5)}$$

Milne's formula is used to 'predict' the value of y_{i+1}which is then used to calculate f_{i+1}

$f_{i+1}=f(x_{i+1}, y_{i+1})$

Then, equation (6) is used to correct the predicted value ofy_{i+1}. The Process is then repeated for the next value of i. Each stage involves four basic calculations, namely

1. Prediction of y_{i+1}
2. evaluation off_{i+1}
3. correction ofy_{i+1}
4. improved value of f_{i+1}(for use in next stage)

It is also possible to use the corrector formula repeatedly to refine the estimate of y_{i+1} before moving on the next stage.

2.2. Adams-Bash forth-Moulton Method

Another popular fourth-order predictor-corrector method is the Adams-Bash forth-Moulton Method multistep method. The predictor formula is known as Adams- Bash forth predictor and is given by

$$y_{i+1} = y_i+\frac{h}{24}(55f_i-59f_{i-1}+37f_{i-2}-9f_{i-3}) \quad \text{............ (6)}$$

The corrector formula is known as Adams-Moulton Corrector and is given by

$$y_{i+1} = y_i+\frac{h}{24}(f_{i-2}-5f_i+19f_{i-1}-9f_{i+1}) \quad \text{............ (7)}$$

This pair of equations can be implemented using the procedure described for Milne-Simpson *method*.

TAYLOR SERIES METHOD

In mathematics, a Taylor Series is a representation of a function as an infinite sum of terms that are calculated from the values of the functions derivatives at a single point

The numerical solution of the equation

$\frac{dy}{dx}$=f(x, y)

Given the initial condition y (x_0) = y_0

y_1=y (x_1)= $y_0+\frac{h}{1!}y_0{}'+\frac{h^2}{2!}y_0{}''+$....................

EULER'S METHOD

Euler's method is the simplest one-step method and has a limited application because of its low accuracy. However, it is discussed here as it serves as a starting point for all other advanced methods.

Given the differential equation

$\frac{dy}{dx}$=f(x, y)

Given the initial condition y (x_0) = y_0

We have $\frac{dy}{dx}=f(x_0, y_0)$

And therefore Y(x) =y (x_0)+ (x -x_0) $f(x_0, y_0)$

Then the value of y(x) at x=x_1 is given by

Y (x_1) = y (x_1)+ $(x_1 - x_0)$ $f(x_0, y_0$

Letting h=x_1 -x_0, we obtain $y_1=y_0+hf(x_0, y_0)$

Similarly, y(x) at x=x_2 is given by

$y_2=y_1+hf(x_1, y_1)$

In general, we obtain a recursive relation as

$y_{n+1}=y_n+hf(x_n, y_n)$; n= 0, 1, 2...............

Improved Euler method

Let the tangent at(x_0, y_0) to the curve be p_0A. In the interval(x_0, x_1), by previous Euler's method, we approximate the curve by the tangent p_0A.

$y_1{}^{(1)}=y_0+hf(x_0, y_0)$ Where $y_1{}^{(1)}=M_1Q_1$

$Q_1(x_1, y_1{}^{(1)})$. Let Q_1 C be line at Q_1 whose slope is f $(x_1, y_1{}^{(1)})$. Now take the average of the slopes at P_0 and Q_1 i.e.

$\frac{1}{2}[f(x_0, y_0)+f(x_1, y_1{}^{(1)})]$

Now draw line p_0D through P_0 (x_0, y_0) with this as the slope.

That is, y-y_0= $\frac{1}{2}[f(x_0, y_0)+f(x_1, y_1{}^{(1)})]$ (x-x_0)

This line intersects x=x_1 at

$y_1=y_0+\frac{1}{2}h[f(x_0, y_0)+f(x_1, y_1{}^{(1)})]$

$y_1=y_0+\frac{1}{2}h[f(x_0, y_0)+f(x_1, y_0+hf(x_0, y_0))]$

Writing generally,

$y_{n+1}=y_n+\frac{1}{2}h[f(x_n, y_n)+f(x_n+h, y_n+h\ (\ ,\ ))]$

RUNGE-KUTTA METHODS

Runge-Kutta method reduces data requirements to reach the same precision without higher derivative calculation.

The numerical solutions of the numerical equation

$\frac{dy}{dx}$=f(x, y)

Given the initial condition y (x_0) $=y_0$the fourth order Runge-Kutta method algorithm is mostly used in problems unless otherwise mentioned.

$k_1=hf(x, y)$

$k_2=hf(x+\frac{1}{2}h, y+\frac{1}{2}k_1)$

k_3=$h\,f(x+\frac{1}{2}h, y+\frac{1}{2}k_2)$
k_4=$h\,f(x+\frac{1}{2}h, y+\frac{1}{2}k_3)$
Δy=$\frac{1}{6}(k_1+2k_2+2k_3+k_4)$

EXAMPLE
1. Determine the value of y (0.4) using Milne's method given y'=x+y, y (0) =1 **use Taylor series** to get the values of y (0.1), y (0.2) and y (0.3).

Solution. Here x_0=0, y_0=1, x_1=0.1, x_2=0.2, x_3=0.3, x_4=0.4

$y' = x + y$	y_0'=x_0+y_0 =0+1 =1
y''= 1+y'	y_0''=1+ y_0' =1+1 =2
y'''=y''	y_0'''=y_0'' =2

Y (0.1) = 1+ (0.1) (1) +0.01+0.0003 =1.1103
Y (0.2) = 1.1103+0.1210+0.0111+0.0003 =1.2427
Y (0.3) =1.2427+0.1443+0.0122+0.0004 =1.3996
Now, knowing y_0, y_1, y_2, y_3 we will find y_4.

By Milne's predictor formula
$y_{4,p}$=1+0.1333[2.4206-1.4427+3.3992]
y_1'=x_1+y_1 =0.1+1.1103 =1.2103
y_2'=x_2+y_2 =0.2+1.2427 =1.4427
y_3'=x_3+y_3 =0.3+1.3996 =1.6996
$y_{4,p}$=1.5835
$y_{4,p}'$=x_4+y_4 =0.4+1.5835 =1.9835

Milne's Corrector formula
$y_{4,c}$=1.2427+0. 0333[1.4427+6.7984+1.9835] =y (0.4) =1.5832

Adam's predictor formula
$y_{4,p}$= 1.3996+0.0042[93.478-85.1193+44.781-9] =1.5849
$y_{4,p}'$=x_4+y_4 =0.4+1.5849 =1.9849

Adam's corrector formula
$y_{4,c}$= 1.3996+0.0042[17.8641+32.2924-7.2135+1.2103] =y (0.4) =1.5851 2. Determine the value of y (0.4) using Milne's method given y'=x+y, y (0) =1 **use Euler's method** to get the values of y (0.1), y (0.2) and y (0.3).

y_{n+1}=y_n+h$f(x_n, y_n)$
y_1= 1+ (0.1) f(0,1)= Y (0.1) =1.1
y_2= 1.1+ (0.2) f((0.1), (1.1)= Y (0.2) =1.22
y_3= 1.22+ (0.1) $f(0.2 + 1.22)$= Y (0.3) =1.362

By Milne's predictor formula
$y_{4,p}$=1+0.1333[2.4-1.42+3.324] =1.5737
y_1'=x_1+y_1 =0.1+1.1 =1.2
y_2'=x_2+y_2 =0.2+1.22 =1.42
y_3'=x_3+y_3 =0.3+1.362=1.662
$y'_{4,p}$ =1.9737

Milne's corrector formula
$y_{4,c}$=1.22+0.0333[1.42+6.648+1.9737] = y (0.4) =1.5544

By Adam's predictor formula
$y_{4,p}$ =1.362+0.0042[91.41-83.78+44.4-9] =1.5427
$y'_{4,p}$ =1.9427

Adam's corrector formula
$y_{4,c}$=1.362+0.0042[17.4843+31.578-7.1+1.2] =y (0.4) =1.5433

3. Determine the value of y (0.4) using Milne's method given y'=x+y, y (0) =1 **use Improved Euler's method** to get the values of y (0.1), y (0.2) and y (0.3).

y_{n+1}=y_n+ $\frac{1}{2}h[f(x_n, y_n) + f(x_n + h, y_n$+h $f(x_n, y_n)]$

y_1=1+ $\frac{1}{2}$(0.1) [f(0, 1) + f(0+0.1, 1+0.1 f(0, 1)] = Y (0.1) =1.11

y_2=1.11+ $\frac{1}{2}$(0.1) [f(0.1, 1.11) + f(0.1+0.1, 1.11+0.1 f(0.1, 1.11)] = Y (0.2) =1.2421

y_3=1.2421+ $\frac{1}{2}$(0.1) [f(0.2, 1.2421) + f(0.2+0.1, 1.2421+0.1 f(0.2, 1.2421)] = Y (0.3) =1.3985

By Milne's predictor formula
$y_{4,p}$=1+0.1333[2.42-1.4421+3.397]
y_1'=x_1+y_1 =0.1+1.11 =1.21
y_2'=x_2+y_2 =0.2+1.2421 =1.4421
y_3'=x_3+y_3 =0.3+1.3985 =1.6985
$y_{4,p}$=1.5832
$y_{4,p}'$=x_4+y_4 =0.4+1.5835 =1.9832

Milne's Corrector formula
$y_{4,c}$=1.2421+0. 0333[1.4421+6.794+1.9832] =y (0.4) =1.5827

Adam's predictor formula
$y_{4,p}$= 1.3985+0.0042[93.4175-85.0839+44.77-9] =1.5837
$y_{4,p}'$=x_4+y_4 =0.4+1.5849 =1.9837

Adam's corrector formula
$y_{4,c}$= 1.3985+0.0042[17.8533+32.2715-7.2105+1.21] =y (0.4) =1.5838

4. Determine the value of y (0.4) using Milne's method given y'=x+y, y (0) =1 **use Runge-Kutta method** to get the values of y (0.1), y (0.2) and y (0.3).

k_1= (0.1) (0+1) =0.1
k_2= (0.1) (0.05+1.05) =0.11
k_3= (0.1) (0.05+1.055) =0.1105
k_4= (0.1) (0.1+1.1105) =0.12105
Δy=0.11034
Y (0.1) =1.11034

k_1= (0.1) (0.1+1.1103) =0.1210
k_2= (0.1) (0.05+1.05) =0.1321
k_3= (0.1) (0.05+1.055) =0.13264
k_4= (0.1) (0.2+1.1105) =0.1443
Y (0.2) =1.2428

k_1= (0.1) (0.2+1.2428) =0.1443
k_2= (0.1) (0.25+1.3149) =0.1565
k_3= (0.1) (0.25+1.3211) =0.1571
k_4= (0.1) (0.4, +1.3999) =0.1799
Y (0.3) =1.4014

By Milne's predictor formula

$y_{4,p}$=1+0.1333[2.4207-1.4428+3.4028]

$y_1'=x_1+y_1$ =0.1+1.11034 =1.2104

$y_2'=x_2+y_2$ =0.2+1.2428 =1.4428

$y_3'=x_3+y_3$ =0.3+1.4014 =1.7014

$y_{4,p}$=1.5839

$y_{4,p}'=x_4+y_4$ =0.4+1.5835 =1.9839

Milne's Corrector formula

$y_{4,c}$=1.2428+0.0333[1.4428+6.8056+1.9839] = y (0.4) =1.5835

Adam's predictor formula

$y_{4,p}$= 1.4014+0.0042[93.577-85.1252+44.78258-9] =1.5872

$y_{4,p}'=x_4+y_4$ =0.4+1.5872 =1.9872

Adam's corrector formula

$y_{4,c}$= 1.4014+0.0042[17.8848+32.3266-7.214+1.21034] =y (0.4) =1.5871.

X		Taylor's series	Euler's method	Improved Euler's method	Runge-Kutta method	Exact Solution
0		1	1	1	1	1
0.1		1.11034	1.1	1.11	1.11034	1.1103
0.2		1.2427	1.22	1.2421	1.2428	1.2428
0.3		1.3996	1.362	1.3985	1.4014	1.3997
0.4	Milne's	1.5849	1.5544	1.5827	1.5835	1.5836
	Adams	1.5851	1.5433	1.5838	1.5871	1.5836

Error Value

X		Taylor's series	Euler's method	Improved Euler's method	Runge-Kutta method
0		0	0	0	0
Ss 0.1		-0.00004	0.0103	0.0003	-0.00004
0.2		0.0001	0.0228	0.0007	0
0.3		0.0001	0.0377	0.0012	-0.0017
0.4	Milne's	-0.0013	0.0292	0.0009	0.0001
	Adams	-0.0015	0.0403	-0.0002	-0.0035

CONCLUSION

In this paper, we compared the various numerical method with the Adam's predictor and corrector and Milne's predictor and corrector. The problem of **Runge-Kutta method** has the minimum error value. So the **Runge-Kutta method** problem is best to approximate and exact solution.

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