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Comparison of Gauss Jacobi Method and Gauss Seidel Method using Scilab

S. Sathya[1], T. Ramesh[2]

[1]M.Sc Mathematics, [2]Assistant Professor,
[1,2]Department of Mathematics, Dr. SNS Rajalakshmi College of Arts and Science, Coimbatore, Tamil Nadu, India

ABSTRACT
Numerical Method is the important aspects in solving real world problems that are related to Mathematics, science, medicine, business etc. In this paper, We comparing the two methods by using the scilab 6.0.2 software coding to solve the iteration problem. which are Gauss Jacobi and Gauss Seidel methods of linear equations.

KEYWORDS: *Linear equation, Gauss Jacobi Method, and Gauss Seidel Method*

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INTRODUCTION

A Linear System of variables $x_1,x_2,......x_n$ is linear equations of the form $a_1x_1+a_2x_2+........a_nx_n= b_n$.The value of $a_1,a_2......a_n$ are any constant real/complex numbers. The constant a_1 is called the coefficient of x_1 and b is called the constant term of the equation.A system of linear equations (or linear system) is a finite collection of linear equation in same variables.

$$a_{11}x_1+a_{12}x_2+........a_{1n}x_n= b_1$$

$$a_{21}x_1+a_{22}x_2+........a_{2n}x_n= b_2$$

$$\cdots\cdots\cdots$$

$$a_{n1}x_1+a_{n2}x_2+........a_{nn}x_n= b_n$$

A solution of a linear system is n- tuple $(v_1, v_2,........v_n)$ of numbers that satisfied each linear equation when the values $v_1,v_2,........v_n$ are substituted for $x_1,x_2,........x_n$ respectively .

There are two Iterative methods for the solving simultaneous equations.

1. **Gauss Jacobi Method**
2. **Gauss Seidel Method**

It can be shown that the Gauss-Seidel method converges twice as fast as Jacobi method.

METHODS

1. Gauss Jacobi Method

Let us explain this method is the case of linear equations in three unknowns.

Consider the system of linear equations,

$$a_1x+b_1y+c_1z= d_1$$
$$a_2x+b_2y+c_2z= d_2$$
$$a_3x+b_3y+c_3z= d_3$$

For converges $|a_1|> |b_1| + |c_1|$
$|b_2|> |a_2| + |c_2|$
$|c_3|> |a_3| + |b_3|$

For iterative process

$$x=\frac{1}{a_1}(d_1- b_1y- c_1z)$$
$$y=\frac{1}{b_2}(d_2- a_2x- c_2z)$$
$$z=\frac{1}{c_3}(d_3- a_3x- b_3z)$$

If $x^{(0)},y^{(0)},z^{(0)}$ are the initial values of x,y,z respectively, then

$$x^{(1)}=\frac{1}{a_1}(d_1- b_1y^{(0)}- c_1z^{(0)})$$
$$y^{(1)}=\frac{1}{b_2}(d_2- a_2x^{(0)}- c_2z^{(0)})$$
$$z^{(1)}=\frac{1}{c_3}(d_3- a_3x^{(0)}- b_3z^{(0)})$$

Again using these value $x^{(1)}$,$y^{(1)}$,$z^{(1)}$ we get

$$x^{(2)}=\frac{1}{a_1}(d_1 - b_1y^{(1)} - c_1z^{(1)})$$
$$y^{(2)}=\frac{1}{b_2}(d_2 - a_2x^{(1)} - c_2z^{(1)})$$
$$z^{(2)}=\frac{1}{c_3}(d_3 - a_3x^{(1)} - b_3z^{(1)})$$

Proceeding in the same way , if the rth iterates are $x^{(r)}$,$y^{(r)}$,$z^{(r)}$, the iteration scheme reduces to

$$x^{(r+1)}=\frac{1}{a_1}(d_1 - b_1y^{(r)} - c_1z^{(r)})$$
$$y^{(r+1)}=\frac{1}{b_2}(d_2 - a_2x^{(r)} - c_2z^{(r)})$$
$$z^{(r+1)}=\frac{1}{c_3}(d_3 - a_3x^{(r)} - b_3z^{(r)})$$

The procedure is continued till the convergence is assured (correct to required decimals).

PROBLEM: Solve the following equation by Gauss Jacobi Method.

10x-5y-2z=3; 4x-10y+3z=-3; x+6y+10z=-3.

Solution:

The below result is find out by the manual work:

n	X	y	z
1	0.3	0.3	-0.3
2	0.39	0.33	-0.51
3	0.363	0.303	-0.537
4	0.3441	0.2841	-0.5181
5	0.33843	0.2822	-0.50487
6	0.340126	0.283911	-0.503163
7	0.3413229	0.2851015	-0.5043592
8	0.34167891	0.2852214	-0.50519319
9	0.341572062	0.285113607	-0.505300731

By using Scilab coding the result shown below:

finallgaussjacobi.sci (C:\Users\Admin\Documents\finallgaussjacobi.sci) - SciNotes

```
clc;clear;close;

x0=0;y0=0;z0=0;
deff('x=f1(y,z)','x=(3+5*y+2*z)/10')
deff('y=f2(x,z)','y=(-3-4*x-3*z)/-10')
deff('z=f3(x,y)','z=(-3-x-6*y)/10')
for i=1:9
    x1=f1(y0,z0);
    y1=f2(x0,z0);
    z1=f3(x0,y0);
    printf('\tx(%i)= %g\n\n\ty(%i)= %g\n\n\tz(%i)=%g\n\n\n\n',i,x1,i,y1,i,z1)
    x0=x1;y0=y1;z0=z1;
end
printf('Thus we find that solution converges to %g,%g and %g',x0,y0,z0)
```

n	X	y	z
1	0.3	0.3	-0.3
2	0.39	0.33	-0.51
3	0.363	0.303	-0.537
4	0.3441	0.2841	-0.5181
5	0.33843	0.28221	-0.50487
6	0.340131	0.283911	-0.503169
7	0.341322	0.285102	-0.50436
8	0.341679	0.285221	-0.505193
9	0.341572	0.285114	-0.5053

The Actual Values are **x=0.3415, y=0.2852, and z=−0.5053.**

2. Gauss-Seidel Method

This is only a refinement of Gauss-Jacobi Method As before,

$$x=\frac{1}{a_1}(d_1 - b_1y - c_1z)$$
$$y=\frac{1}{b_2}(d_2 - a_2x - c_2z)$$
$$z=\frac{1}{c_3}(d_3 - a_3x - b_3y)$$

We start with the initial values $y^{(0)}$,$z^{(0)}$ for y and z and get $x^{(1)}$ That is,

$$x^{(1)}=\frac{1}{a_1}(d_1 - b_1y^{(0)} - c_1z^{(0)})$$

We use $z^{(0)}$ for z and $x^{(1)}$ for x instead of $x^{(0)}$as in the Jacobi's Method, we get

$$y^{(1)}=\frac{1}{b_2}(d_2 - a_2y^{(1)} - c_2z^{(0)})$$

Now , having known $x^{(1)}$and $y^{(1)}$ use $x^{(1)}$ for x and $y^{(1)}$for y ,we get

$$z^{(1)}=\frac{1}{c_3}(d_3 - a_3x^{(1)} - b_3y^{1})$$

In finding the values of the unknowns, we use the latest available values on the right hand side. If $x^{(r)}$,$y^{(r)}$,$z^{(r)}$ are the rth iterates , then the iterates scheme will be

$$x^{(r+1)}=\frac{1}{a_1}(d_1 - b_1y^{(r)} - c_1z^{(r)})$$
$$y^{(r+1)}=\frac{1}{b_2}(d_2 - a_2x^{(r+1)} - c_2z^{(r)})$$
$$z^{(r+1)}=\frac{1}{c_3}(d_3 - a_3x^{(r+1)} - b_3y^{(r+1)})$$

This Process of iteration is continued until the convergence is assured.

PROBLEM: Solve the following equation by Gauss-Seidel Method.

10x-5y-2z=3; 4x-10y+3z=-3; x+6y+10z=-3.

Solution:

The result is find out by the manual work.

N	x	y	z
1	0.3	0.42	-0.582
2	0.3936	0.28284	-0.509064
3	0.3396072	0.28312364	-0.503834928
4	0.34079485	0.28516746	-0.50517996
5	0.3415547	0.28506792	-0.505196229
6	0.3414947	0.285039	-0.5051728
7	0.3414849	0.28504212	-0.5051737

By using the Scilab coding the result is shown below:

finalgaussseidel.sci (C:\Users\Admin\Documents\finalgaussseidel.sci) - SciNotes

```
clc;clear;close;

x0=0;y0=0;z0=0;
deff('x=f1(y,z)','x=(3+5*y+2*z)/10')
deff('y=f2(x,z)','y=(-3-4*x-3*z)/-10')
deff('z=f3(x,y)','z=(-3-x-6*y)/10')
for i=1:7
    x0=f1(y0,z0);
    y0=f2(x0,z0);
    z0=f3(x0,y0);
    printf('\tx(%i)=%g\n\n\ty(%i)=%g\n\n\tz(%i)=%g\n\n\n\n',i,x0,i,y0,i,z0)
end
printf('thus we find that solution converges to %g,%g and %g',x0,y0,z0)
```

N	x	y	z
1	0.3	0.42	-0.582
2	0.3936	0.28284	-0.509064
3	0.33960	0.283124	-0.503835
4	0.340795	0.285167	-0.50518
5	0.341548	0.285065	-0.505194
6	0.341494	0.285039	-0.505173
7	0.341485	0.285042	-0.505174

The Actual value are **x=0.3415, y=0.2852, and z= -0.5053.**

CONCLUSION

There are different methods of solving of linear equation some are direct methods while some are iterative methods. In this paper , two Iteration methods of solving of linear equation have been presented where the Gauss-Seidel Method proved to be the best and effective in the sense that it converge very fast with Scilab Software.

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