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Foundation and Synchronization of the Dynamic Output Dual Systems

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ABSTRACT

In this paper, the synchronization problem of the dynamic output dual systems is firstly introduced and investigated. Based on the time-domain approach, the state variables synchronization of such dual systems can be verified. Meanwhile, the guaranteed exponential convergence rate can be accurately estimated. Finally, some numerical simulations are provided to illustrate the feasibility and effectiveness of the obtained result.

KEYWORDS: *Dynamic output dual systems, time-domain approach, hyper-chaotic system; exponential synchronization, exponential convergence rate*

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1. INTRODUCTION:

In recent years, various types of chaotic systems have been widely explored and studied by experts and scholars; see, for instance, [1-6] and the references therein.

This is not only due to the mystery of its theory, but also to its various applications in dynamic systems. As we know, chaos is often one of the factors that cause system instability and oscillation.

In the past decades, the synchronization design of various systems has been developed and explored and has quite good results; see, for instance, [7-11] and the references therein. Moreover, synchronization design frequently exists in various fields of application, such as master-slave chaotic systems, system identification, ecological systems, and secure communication. Due to the unpredictability of chaotic systems, this feature becomes more challenging for the synchronization design of chaotic dual systems.

On the other hands, a variety of dual systems have been proposed, investigated and studied in depth; see, for instance, [12-14]. A variety of methodologies have been proposed for analyzing dual systems, such as separation principle, passivation of error dynamics, sliding-mode approach, frequency domain analysis, and Chebyshev neural network (CNN). In particular, the synchronization analysis and design of dynamic systems with chaos is in general not as easy as that without chaos. On the basis of the above-mentioned reasons, the synchronization analysis and design of chaotic dual systems is actually crucial and meaningful.

In this paper, the synchronization problem for a class of chaotic dual systems will be considered. Based on the time-domain approach with differential inequality, the state variables synchronization of such dual systems will be verified. In addition, the guaranteed exponential convergence rate can be accurately estimated. Several numerical simulations will also be provided to illustrate the use of the obtained results. In what follows, $\|x\|$ denotes the Euclidean norm of the vector $x \in \Re^n$.

2. PROBLEM FORMULATION AND MAIN RESULTS

In this paper, we consider the following master-slave dual systems.

Master chaotic system:

$$\begin{cases} \dot{x}_1(t) = a\,x_2(t) + f_1(x_1), \\ \dot{x}_2(t) = f_2(x_1(t), x_2(t)) + bx_1(t)x_3(t) \\ \qquad +cx_4(t), \\ \dot{x}_3(t) = -dx_3(t) + f_3(x_1(t), x_2(t)), \\ \dot{x}_4(t) = f_4(x_1(t), x_2(t), x_3(t), x_4(t)), \\ y(t) = \beta x_1(t), \quad \forall\, t \ge 0, \end{cases} \tag{1}$$

Slave system:

$$\begin{cases} z_1(t)=\dfrac{y(t)}{\beta}, \\ z_2(t)=\dfrac{1}{a\beta}\dot{y}(t)-\dfrac{1}{a}f_1(z_1(t)), \\ \dot{z}_3(t)=-d\,z_3(t)+f_3(z_1(t),z_2(t)), \\ z_4(t)=\dfrac{1}{c}[\dot{z}_2(t)-f_2(z_1(t),z_2(t))-bz_1(t)z_3(t)], \\ \qquad \forall\, t\ge 0. \end{cases} \quad (2)$$

where $x(t):=[x_1(t)\quad x_2(t)\quad x_3(t)\quad x_4(t)]^T\in\Re^4$ is the state vector of master chaotic system, $y(t)\in\Re$ is the system output, $z(t):=[z_1(t)\quad z_2(t)\quad z_3(t)\quad z_4(t)]^T\in\Re^4$ is the state vector of slave system, a,b,c,d,β are the system parameters with $d>0$ and $ac\beta\neq 0$. For the existence and uniqueness of the system (1), we assume that all the functions $f_i(\cdot),\ \forall\, i\in\{1,2,3,4\}$, are sufficiently smooth.

Remark1. It is noted that the modified hyper-chaotic Pan system [1] is the special cases of system (1) with

$$a=10,\quad b=-1,\quad c=1,\quad d=\frac{8}{3},$$
$$f_1(x_1)=-10x_1,\quad f_2(x_1,x_2)=28x_1,$$
$$f_3(x_1,x_2)=x_1x_2,\quad f_4(x_1,x_2,x_3,x_4)=-10x_2.$$

The time response of the modified hyper-chaotic Pan system is depicted in Fig. 1-Fig. 4.
For brevity, let us define the synchronous error vector as

$$e(t):=[e_1(t)\quad e_2(t)\quad e_3(t)\quad e_4(t)]^T := x(t)-z(t) \quad (3)$$

The precise definition of exponential synchronization is given as follows.

Definition1. The slave system (2) exponentially synchronizes the master chaotic system (1) provided that there are positive numbers k and α such that
$\|e(t)\|\le k\exp(-\alpha t),\quad \forall\, t\ge 0$.

In this case, the positive number α is called the exponential convergence rate.

Now we present the main result for the master-slave dual systems (1) and (2).

Theorem1. The slave system (2) exponentially synchronizes the master chaotic system (1). Besides, the guaranteed exponential convergence rate is given by d.

Proof. From (1)-(3), it can be readily obtained that

$$e_1(t)=x_1(t)-z_1(t) = \frac{y(t)}{\beta}-\frac{y(t)}{\beta}=0,\quad \forall\, t\ge 0; \quad (4)$$

$$\begin{aligned} e_2(t)&=x_2(t)-z_2(t) \\ &=\left[\frac{1}{a}\dot{x}_1(t)-\frac{1}{a}f_1(x_1)\right] -\left[\frac{1}{a\beta}\dot{y}(t)-\frac{1}{a}f_1(z_1(t))\right] \\ &=\left[\frac{1}{a}\dot{x}_1(t)-\frac{1}{a}f_1(x_1)\right] -\left[\frac{1}{a}\dot{x}_1(t)-\frac{1}{a}f_1(x_1(t))\right] \\ &=0,\quad \forall\, t\ge 0; \end{aligned} \quad (5)$$

$$\begin{aligned} \dot{e}_3(t)&=\dot{x}_3(t)-\dot{z}_3(t) \\ &=[-dx_3(t)+f_3(x_1,x_2)] -[-d\,z_3(t)+f_3(z_1(t),z_2(t))] \\ &=-d[x_3(t)-z_3(t)]+f_3(x_1,x_2) - f_3(z_1,z_2) \\ &=-de_3(t)+f_3(x_1,x_2)-f_3(x_1,x_2) \\ &=-de_3(t),\quad \forall\, t\ge 0. \end{aligned}$$

This implies

$$\frac{d[e_3(t)\exp(d\,t)]}{dt}=0$$
$$\Rightarrow\ e_3(t)\exp(d\,t)=e_3(0)\exp(0)$$
$$\Rightarrow\ e_3(t)=\exp(-d\,t)e_3(0),\quad \forall\, t\ge 0. \quad (6)$$

In addition, from (1)-(6), one has

$$\begin{aligned} e_4(t)&=x_4(t)-z_4(t) \\ &=\left[\frac{\dot{x}_2(t)}{c}-\frac{f_2(x_1(t),x_2(t))}{c}-\frac{bx_1(t)x_3(t)}{c}\right] -\left[\frac{\dot{z}_2(t)}{c}-\frac{f_2(z_1(t),z_2(t))}{c}-\frac{bz_1(t)z_3(t)}{c}\right] \\ &=\left[\frac{\dot{x}_2(t)}{c}-\frac{f_2(x_1(t),x_2(t))}{c}-\frac{bx_1(t)x_3(t)}{c}\right] -\left[\frac{\dot{x}_2(t)}{c}-\frac{f_2(x_1(t),x_2(t))}{c}-\frac{bx_1(t)z_3(t)}{c}\right] \\ &=-\frac{bx_1(t)e_3(t)}{c} \\ &=-\frac{bx_1(t)\exp(-d\,t)e_3(0)}{c},\quad \forall\, t\ge 0. \end{aligned} \quad (7)$$

As a result, from (4)-(7), we conclude that

$$\|e(t)\|=\sqrt{e_1^2(t)+e_2^2(t)+e_3^2(t)+e_4^2(t)} \le k\cdot\exp(-d\,t),\quad \forall\, t\ge 0,$$

in view that $x_1(t)$ is a chaotic signal. This completes the proof.

By Theorem 1 with Remark 1, it is straightforward to obtain the following result.

Corollary1. The slave system of

$$\begin{cases} z_1(t)=\dfrac{y(t)}{\beta}, \\ z_2(t)=\dfrac{1}{10\beta}\dot{y}(t)+z_1(t), \\ \dot{z}_3(t)=-\dfrac{8z_3(t)}{3}+z_1(t)z_2(t), \\ z_4(t)=\dot{z}_2(t)-28z_1(t)+z_1(t)z_3(t),\quad \forall\, t\ge 0, \end{cases}$$

exponentially synchronizes the modified hyper-chaotic Pan system with the guaranteed exponential convergence rate is given by $\alpha=\dfrac{8}{3}$.

3. NUMERICAL SIMULATIONS
Consider the slave system:

$$\begin{cases} z_1(t)=\dfrac{y(t)}{2}, \\ z_2(t)=\dfrac{1}{20}\dot{y}(t)+z_1(t), \\ \dot{z}_3(t)=-\dfrac{8z_3(t)}{3}+z_1(t)z_2(t), \\ z_4(t)=\dot{z}_2(t)-28z_1(t)+z_1(t)z_3(t), \quad \forall t\geq 0. \end{cases} \quad (8)$$

By Corollary 1, we conclude that the system (8) is exponentially synchronizes the following master chaotic system

$$\begin{cases} \dot{x}_1(t)=10x_2-10x_1, \\ \dot{x}_2(t)=28x_1-x_1x_3+x_4, \\ \dot{x}_3(t)=\dfrac{-8}{3}x_3+x_1x_2, \\ \dot{x}_4(t)=-10x_2, \\ y(t)=2x_1, \quad \forall t\geq 0, \end{cases} \quad (9)$$

with the guaranteed exponential convergence rate $\alpha=\frac{8}{3}$.

The time response of error states for the systems (8) and (9) is depicted in Fig. 5. From the foregoing simulations results, it is seen that the system (8) is exponentially synchronizes the master system of (9).

4. CONCLUSION
In this paper, the synchronization problem of the dynamic output dual systems has been firstly introduced and investigated. Based on the time-domain approach, the state variables synchronization of such dual systems can be verified. Besides, the guaranteed exponential convergence rate can be accurately estimated. Finally, some numerical simulations have been offered to show the feasibility and effectiveness of the obtained result.

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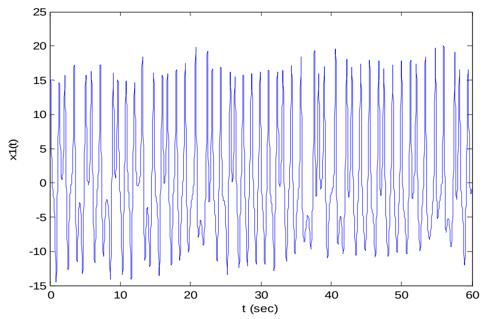

Figure 1: The chaotic signals of $x_1(t)$.

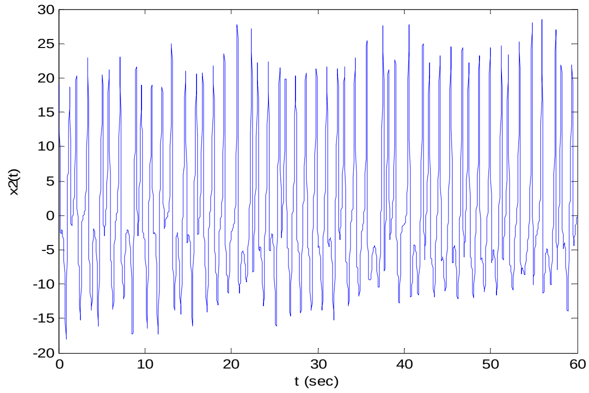

Figure 2: The chaotic signals of $x_2(t)$.

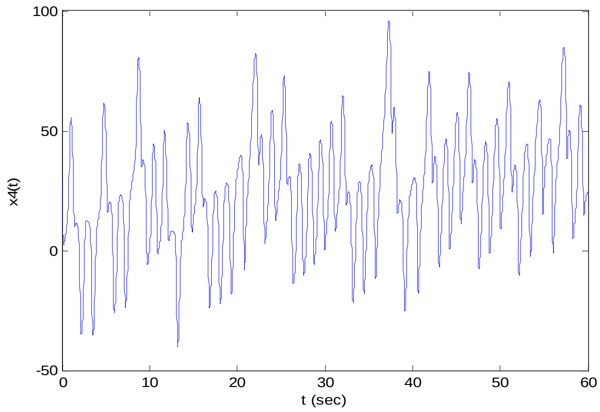

Figure 4: The chaotic signals of $x_4(t)$.

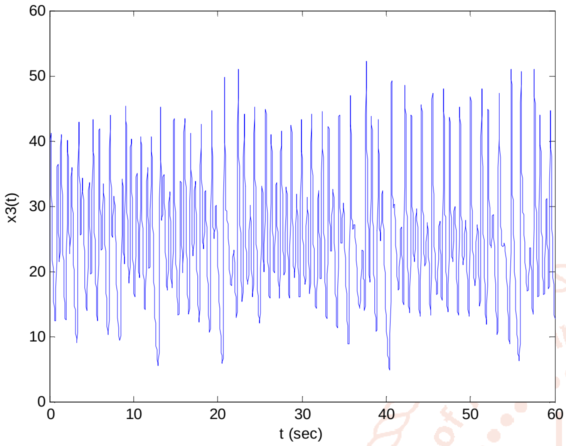

Figure 3: The chaotic signals of $x_3(t)$.

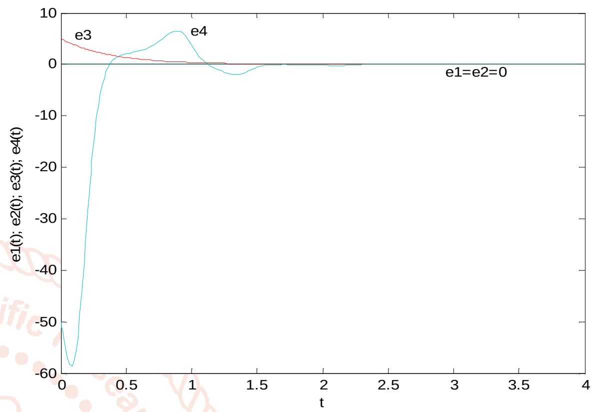

Figure 5: The time response of error states.