International Journal of Trend in Scientific Research and Development(IJTSRD)
Volume 3 Issue 5, August 2019 Available Online: www.ijtsrd.com e-ISSN: 2456-6470

RESULTS ON CHARACTERISTIC VECTORS

M M Jariya
G.A.S.C.BAVLA.
E−mail: *mahesh.jariya@gmail.com*

Abstract

In this paper we have established the bounds of the extreme characteristic roots of nlap(G) and sLap(G) by their traces. Also found the bounds for n-th characteristic roots of nLap(G) and sLap(G).

Key words : characteristic root, Normalized laplacian matrix, signless laplacian matrix.
AMS **subject classification number :** 05C78.

1. Introduction

let G=(V,E) be a graph with $|v| = n$ and $|E| = e$.

k_i denote the degree of u_i for $u_i \in V$

m_i is the set of neighbours of u_i

$i \sim j$ denote the adjacent of u_i and u_j.

Adj(G) denote the adjacency matrix.

Lap(G) denote the laplacian matrix.

dig(G) denote the diagonal matrix.

It is clear $Lap(G) = dig(G) - Adj(G)$

Definition−1 Normalized Laplacian matrix:

let G=(V,E) be a graph then the Normalized laplacian matrix of G is denoted by nLap(G) and given by

nLap(G)= $[r_{ij}]_{mm}$ where

$$\begin{aligned} r_{ij} &= 1 : i = j \\ &= \frac{-1}{\sqrt{k_i k_j}} : i \sim j \\ &= 0 : \text{otherwise} \end{aligned}$$

Definition−2 Signless Laplacian matrix:

let G=(V,E) be a graph then the Signless laplacian matrix of G is denoted by sLap(G) and given by

sLap(G)= $[s_{ij}]_{mm}$ where

$$\begin{aligned} s_{ij} &= k_i : i = j \\ &= 1 : i \sim j \\ &= 0 : \text{otherwise} \end{aligned}$$

Since nlap(G) is real symmetric matrix and characteristic roots of nlap(G) is denoted by μ_1(nlap(G)) $\geq \cdots \geq \mu_m(nlap(G))$ and

slap(G) is also real symmetric matrix and characteristic roots of slap(G) is denoted by μ_1(nlap(G)) $\geq \cdots \geq \mu_m(nlap(G))$.

Following theorem−1.1 and theorem−1.2 are used to find bounds for characteristic roots of nlap(G) and slap(G).

Theorem−1.1 Let V and $\mu = [\mu_j]$ be non zero column vectors, e=(1,1,1,. . . ,1)^T, n=$\frac{\mu^T e}{m}$, B=$I_m - \frac{ee_T}{m}$, $s^2 = \frac{\mu^T B\mu}{m}$ and I_m is a unit matrix. Let $\mu_m \leq \cdots \leq \mu_2 \leq \mu_1$ then,

$$-s\sqrt{mV^T BV} \leq V^T\mu - mV^T e = V^T B\mu \leq s\sqrt{mV^T BV}$$

$$\sum(\mu_i - \mu_m)^2 = m[s^2 + (n - \mu_m)^2]$$

$$\sum(\mu_1 - \mu_j)^2 = m[s^2 + (\mu_1 - \mu_n)^2]$$

$$\mu_m \leq n - \frac{s}{\sqrt{m-1}} \leq n + \frac{s}{\sqrt{m-1}} \leq \mu_1.$$

Theorem−1.2 Let B be m×m with complex entries. A^+ is conjugate transpose of A. Let $B = AA^+$ whose characteristic roots are $\mu_m(C) \leq \cdots \leq \mu_2(C) \leq \mu_1(C)$ Then

$$n - s\sqrt{m-1} \leq \mu_m^2(C) \leq n - \frac{s}{\sqrt{m-1}}.$$

$$n + \frac{s}{\sqrt{m-1}} \leq \mu_m^2(1) \leq n + s\sqrt{m-1}.$$

Where $\frac{Trace(c^2)}{m} - n = s^2$ and $\frac{Trace(B)}{m} = n$.

2. Main Results

Theorem−2.1 Let G is simple graph, $nLap(G)$ is normalized laplacian matrix with characteristic roots $\mu_m(nLap(G)) \leq \cdots \leq \mu_2(nLap(G)) \leq \mu_1(nLap(G))$ Then

$$\mu_m(nLap(G)) \leq \sqrt{(1+\frac{2}{m}\sum_{i<j,i\sim j}\frac{1}{k_ik_j}+\sqrt{\frac{trace[lap(G)]^4-mn^2}{m(m-1)}}}$$

$$\mu_1(nLap(G)) \geq \sqrt{(1+\frac{2}{m}\sum_{i<j,i\sim j}\frac{1}{k_ik_j}+\sqrt{\frac{trace[lap(G)]^4-mn^2}{m(m-1)}}}$$

$$\mu_1(nLap(G)) \leq \sqrt{(1+\frac{2}{m}\sum_{i<j,i\sim j}\frac{1}{k_ik_j}+\sqrt{(\frac{trace[lap(G)]^4}{n}-n^2)(m-1)}}$$

Proof : It is obvious

$$Trace(nLap(G))^2 = m+2\sum_{i<j,i\sim j}\frac{1}{k_ik_j} \text{ and}$$

$$Trace(nLap(G))^4 = \sum_{j=1}^{m}(1+\sum_{i\sim j}\frac{1}{k_ik_j})^2+2\sum_{i<j}(\sum_{k\in m_i\cap m_j}\frac{1}{k_k\sqrt{k_ik_j}}-\sum_{i\sim j}\frac{2}{\sqrt{k_ik_j}})^2$$

Since nLap(G) is real symmetric matrix, we found result from theorem−1.2.

Illustration−2.2 Let $G=(V,E)$, $V=\{1,2,3,4,5,6\}$, $E=\{(5,6),(4,5),(3,5),(3,4),(2,6),(2,4),(2,3),(1,5),(1,2)\}$ Then

μ(nlap(G))	[4]	[7] above bound	[6]below bound
1.86	2	1.93	1.34

Theorem−2.3 Let G is simple graph, $sLap(G)$ is normalized laplacian matrix with characteristic roots $\mu_m(sLap(G)) \leq \cdots \leq \mu_2(sLap(G)) \leq \mu_1(sLap(G))$ Then

$$\mu_m(sLap(G)) \leq \sqrt{(1+\frac{2}{m}\sum_{i<j,i\sim j}\frac{1}{k_ik_j}+\sqrt{\frac{trace[lap(G)]^4-mn^2}{m(m-1)}}}$$

$$\mu_1(sLap(G)) \geq \sqrt{(1+\frac{2}{m}\sum_{i<j,i\sim j}\frac{1}{k_ik_j}+\sqrt{\frac{trace[lap(G)]^4-mn^2}{m(m-1)}}}$$

$$\mu_1(sLap(G)) \leq \sqrt{(1+\frac{2}{m}\sum_{i<j,i\sim j}\frac{1}{k_ik_j}+\sqrt{(\frac{trace[lap(G)]^4}{n}-n^2)(m-1)}}$$

Proof : It is obvious

$$Trace(sLap(G))^2 = m+2\sum_{i<j,i\sim j}\frac{1}{k_ik_j} \text{ and}$$

$$Trace(sLap(G))^4 = \sum_{j=1}^{m}(1+\sum_{i\sim j}\frac{1}{k_ik_j})^2+2\sum_{i<j}(\sum_{k\in m_i\cap m_j}\frac{1}{k_k\sqrt{k_ik_j}}-\sum_{i\sim j}\frac{2}{\sqrt{k_ik_j}})^2$$

Since sLap(G) is real symmetric matrix, we found result from theorem−1.2.

Illustration−2.4 Let $G=(V,E)$, $V=\{1,2,3,4,5,6,7\}$, $E=\{(4,6),(4,5),(3,5),(2,3),(1,7),(1,6),(1,5),(1,4),(1,3),(1,2)\}$ Then

μ(slap(G))	[3]	[2]	[1]	[10] above bound	[9] below bound
7.67	9.3	9.7	9.1	7.8	4.58

4 Concluding Remarks :

I have presented results on connecting the bounds of the extreme characteristic roots of normalized laplacian matrix and signless laplacian matrix. and using it found found the bounds for n-th characteristic roots of normalized and signless laplacian matrix.

References

[1] J. Cheeger, A lower bound for the smallest eigenvalue of the Laplacian, in Problems in analysis, (R.C. Gunnig, ed.), Princeton Univ. Press, 1970, pp.195199.

[2] M. Marcus, H. Minc, A survey of matrix theory and matrix inequalities, Allyn and Bacon, Boston, Mass., 1964.

[3] J. A. Gallian, A dynamic survey on graph labeling, *The Electronics Journal of Combinatorics*, 17(2014), ♯DS6.

[4] D. Cvetkovic, Signless Laplacians and line graphs, Bull. Acad. Serbe Sci. Arts, Cl. Sci. Math. Natur., Sci. Math. 131 (2005), No. 30, 8592.

[5] D. Cvetkovic, P. Rowlinson, S. Simic, Eigenspaces of Graphs, Cambridge University Press, Cambridge, 1997.