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RESULTS ON LINEAR ALGEBRA

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Abstract

In this paper we have established the some results as basic of linear algebra. Last theorem which is known as Cayley-Hamilton theorem. The first proof of this theorem is given in 1878.

Key words : vector space, Dimension, Minimal Polynomial

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1. Introduction

Linear algebra has many applications in engineering field. Cayley hamilton theorem is topic of eigen value-eigen vectors.

In 1935 McCoy [3] proved that the matrices A and B have simultaneous triangularization iff for every polynomial p(x, y) of the noncommutative variables x, y the matrix (A, B)[A, B] is nilpotent. Consequently if the McCoy condition (p(A, B)[A, B] is nilpotent for every p(x, y) as above) holds, then A and B have a common eigenvector.

cayley Hamilton theorem is very important topic of linear algebra. The statement of this theorem is given in 1858 but The generalized proof of this theorem is first given in 19th century.

Vector space is the set of objects which satisfying many conditions like identity, inverse etc.

Definition−1.1 Subspace:

Any non empty subset of vector space is call subspace if it is vector space itself under same operations.

Definition−1.1 Characteristic Polynomial:

For any square matrix A, $det(A - \lambda I)$ is known as characteristic polynomial.

Definition−1.2 Dimension:

Number of vectors of basis of vector space is called dimension of vector space.

2. Main Results

Theorem$-$2.1 let W be a vector space over infinite field R, dim(W) = n. for $j = 1, 2, \dots r$, W_j be subspace of W then $W = W_j$ for some j, If $W = \cup_{j=1}^{s}$.

Proof : Let $U(c_1, c_2, \dots, c_m)$ be undemon matrix,

Let $b_1, b_2, \dots, b_m$ be a F-base of W.

Then

$$\alpha_a = B_1 + aB_2 + a^2 B_3 + \cdots a^{n-1} B_n$$

Suppose $a_1, a_2, \dots, a_m$ be distinct elements in R

We have

$$(\alpha a_1, \alpha a_2, \dots, \alpha a_m) = (B_1, B_2, \dots, B_m)\, U(c_1, c_2, \dots, c_m)$$

$(\alpha a_1, \alpha a_2, \dots, \alpha a_m)$ is basis of W as $det(U(c_1, c_2, \dots, c_m)) \neq 0$.

Let $T = \{\alpha_a : a \in R\}$

We are given w_j, for $j = 1, 2, \dots r$ is non trivial subspace of W.

So we can verify $\quad |T \cap w_j| \leq m - 1$

$$|T \cap (\cup_{j=1}^{s} w_j)| = |\cup_{j=1}^{s} (T \cap w_j)| \leq r(m-1)$$

Therefore $r \setminus r \cap (\cup_{j=1}^{s} w_j)$ is infinite and any different m-vectors in the set contribute basis of W.

Proposition$-$2.2 Let U be vector space over infinite field R and $dim(W) = n$. t be linear $trans^m$ of u then $n_u(x) = n_t(x)$ for $w \in W$.

Proof : Clearly $1, t, t^2, \dots t^{m^2}$ are linearly dependent so $d(n_t(x)) \leq m^2$, for $\forall w \in W$

The minimal polynomial $n_u(x)$ of u is a factor of $n_u(x)$.

So for $u_1, u_2, \dots u_r$, $n_t(x) = n_{u_1}(x) n_{u_2}(x) \cdots n_{u_r}(x)$

Let $W_j = \{B \in W, n_{u_j}(t)\beta = 0\}$.

We can verify, $w_1 \cup w_2 \cup \ldots \cup w_r = w$

By theorem$-$1.1, there exist $1 \leq k \leq r$ such that $w = w_k$ which shows that $n_{u_j}(t)\beta = 0$, for $\forall \beta \in W$.

There fore $n_{u_j}(t)$ is zero transformation.

So $n_{u_j} = n_t$.

Theorem−2.3 Let U be n-dimensional vector space over R and t be linear $tran^m$ of U, for any $\gamma \neq 0$, $B \in U$, There exist $\alpha \in U$ such that $n_a(x) = lcm(n_\gamma(x), n_\beta(x))$

Proof : By rearrangement, the minimal polynomial of γ, β with respect to t have following factorization

$$\begin{aligned} n_\gamma(x) &= v_1(x)v_2(x) \\ &= q_1^{r_1} \cdots q_k^{r_k}, q_{k+1}^{r_{k+1}} \cdots q_l^{r_l} \\ n_\beta(x) &= u_1(x)u_2(x) \\ &= q_1^{s_1} \cdots s_k^{s_k}, q_{s+1}^{r_{k+1}} \cdots q_l^{s_l} \end{aligned}$$

We have

c $lcm(n_\gamma(x), n_\beta(x)) = v_1(x)u_2(x)$

$$gcd(v_1(x)u_2(x)) = 1$$

We can verify that the minimal polynomial of $v_2(t)\gamma$ and $u_1(t)\beta$ are $n_{v_{2(t)}\gamma}(x) = v_1(x)$ and $n_{u_{1(t)}\beta}(x) = u_2(x)$

Let $\alpha = v_2(t)\gamma + u_1(\gamma)\beta$ then $v_1(t)u_2(t)a = 0$

$$\Rightarrow n_a(x) \backslash v_1(x)u_2(x) \qquad \cdots\cdots\cdots (a)$$

Conversely from $v_2(t)\gamma = \alpha - u_1(t)\beta$ shows that $n_a(t)n_{u_{1(t)}\beta}(t)(v_2(t)\gamma) = 0$

which proves that $\quad n_{v_{2(t)}\gamma}(x) \backslash n_a(x)n_{u_{1(t)}\beta}(x)$

means $\quad v_1(x) \backslash n_a(x)u_2(x)$

$$\Rightarrow v_1(x)u_2(x) \backslash n_a(x) \qquad \cdots\cdots\cdots (b)$$

From (a) and (b), $n_a(x) = v_1(x)u_2(x)$

$$= lcm(n_\gamma(x), n_\beta(x)).$$

Theorem−2.4 Let $\Delta_t(t)$ is characteristic polynomial of t Then $n_t(t) \backslash \Delta_t(t)$ and $\Delta_t(t) = 0$

Proof : Since $\Delta_t(t)$ is characteristic polynomial of t for any $u \in W$.

Let $n_u(t)$ be minimal polynomial of the vector u w.r. to t. To show given theorem it is enough to prove $n_u(t) \backslash \Delta_t(t)$ for any $u \in W$.

4 Concluding Remarks :

presented results are on basic of linear algebra. In this paper we have derived four new results which are useful in linear algebra. and theorem-2.4 is known as cayley-Hamilton Theorem.

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