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Ant Colony System with Saving Heuristic for Capacitated Vehicle Routing Problem

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ABSTRACT

The ACO heuristics is a distributed and cooperative search method that imitates the behavior of real ants in its the search for food. The Capacitated Vehicle Routing Problem (CVRP) is a well-known combinatorial optimization problem, which is concerned with the distribution of goods between the depot and customers. This paper will apply the Ant Colony System (ACS) with Savings heuristic algorithm to solve Capacitated Vehicle Routing Problem. This problem will be solve to determine an optimal distribution plan that meets all the demands at minimum total cost by applying the ACS algorithm. In this paper, we consider that there is a single depot (or distribution center) that caters to the customer demands at a set of sales points (or demand centers) using vehicles with known limited capacities. The demand at each of these demand centers is assumed to be constant and known. Due to its limited capacity, the vehicles may need to make several trips from the depot for replenishment. This system will implement the transportation cost of CVRP and can find the minimum cost routes between the depot and the customers by using the Benchmarks datasets.

KEYWORDS: *Ant Colony System, Vehicle Routing, Heuristics, Capacitated Vehicle Routing Problem.*

1. INTRODUCTION

Ant Colony Optimization (ACO) is a meta-heuristic algorithm. The ACO simulates the natural ant treatment for food finding and applies it for solving the combinational optimization problem.

Ant Colony System (ACS) is inspired in the foraging behavior of real ants. When searching for food, ants initially explore the area surrounding their nest in a random manner. While moving, ants leave a chemical pheromone trail on the ground. Ants communicate using pheromone trail. Ants deposit pheromone along the path that other ants can follow. ACS uses the pheromone trail as communication medium.

The Vehicle Routing Problem (VRP) is concerned with the design of the optimal routes, used by a fleet of identical vehicles stationed at a central depot to serve a set of customers with known demands. In the basic version of the problem, known as a Capacitated VRP (CVRP), only capacity restriction for vehicles are considered and the objective is to minimize the total cost (or length) of routes. In the CVRP, one has to deliver goods to a set of customers with known demands on minimum-cost vehicle routes originating and terminating at a depot.

CVRP includes not only the optimization of a path, but many of them simultaneously, since a fleet of evenly capacitated vehicles have to deliver goods to geographically distributed customers with variable demand, travelling the least distance as possible. In this paper, vehicles are positioned on customers randomly and initial pheromone trail levels are applied to route. When constructing routes if all remaining choices would result in an infeasible solution due to vehicle capacity being exceeded, then the depot is chosen and a new route is started. Vehicles will choose the next customer to visit using a combination of heuristic and pheromone information. During the construction of a route, the algorithm modifies the amount of pheromone on the chosen route by applying a local updating rule. Once all vehicles have constructed their tours then the amount of pheromone on routes belonging to the best solution, as well as the global best solution, updated according to the global updating rule. Finally, the system will show the result minimum cost routes and minimum number of vehicles.

The remainder of this paper will organize as follow. Section-2 describes the related works, section-3 explains materials and method and section-4 describes the system overview and architecture. This paper will conclude in section-5.

2. RELATED WORKS

A traditional business model is articulated in three stages: production, distribution, and sales. Each one of these activities is usually managed by a different company, or by a different branch of the same company [2]. Dorigo and Stutzle (2004) first introduced the Ant Colony Optimization (ACO) that is inspired by the real life behavior of ants foraging for food. During the search for food from their nest to the food source, it was found that a moving ant will lay a chemical substance called pheromone on the trail. The pheromone trail is a form of communication among the ants, which will attract the other ants to use the same path to travel. Thus, higher amount of pheromone will enhance the probability of the next ant selecting that path to travel. With times, as more ants are able to complete the shorter path, the pheromone will accumulate faster on shorter path compared to the longer path. Consequently, majority of the ants would have travelled on the shortest path. Detailed descriptions of the ACO can be found in the book by Dorigo and Stutzle (2004). One of the most efficient ACO based implementations has been Ant Colony System (ACS)[10], that introduced a particular pheromone trail up-dating procedure useful to intensify the search in the neighborhood of the best computed solution.

Many researchers have proposed new methods to improve the original ACO especially by applying other algorithms into the ACO to tackle the large-scaled the Capacitated Vehicle Routing Problem (CVRP). For instance, Doerner *et al.* (2002) [6] proposed a hybrid approach for solving the CVRP by combining the AS with the savings algorithm. After that, Reimann *et al.* (2002) [15] improved on the method in [6] by presenting a Savings based Ant System (SbAS) and then Reimann *et al.* (2004) [14] proposed an approach called D-Ants which is competitive with the best Tabu Search (TS) algorithm in terms of solution quality and computation time.

Then, CVRP problem is present in real-world organizations, such as hauliers and goods distributors. There are many daily tasks that can be modeled as this kind of problem, for instance: trash collection, school transportation and mail home delivery [11]. It is of economic importance to businesses as the transportation process contributes approximately 10-20% of the final cost of the goods. The author Lee, L.S., [16] present the CVRP problem using an Ant Colony Optimization (ACO) combined with heuristic approaches. In which, the algorithms shown that the application of combination of two different heuristics in the ACO had the capability to improve the ants' solutions better than ACO embedded with only one heuristic and compared the solution quality of different basic heuristics combined with an original ACO in solving the problem. The computational results of the problems shown that the ACO combined with the heuristic approach has the capability to tackle the CVRP with satisfactory solution.

3. Material and methods

The ACO heuristics is a distributed and cooperative search method that imitates the behavior of real ants in its the search for food. The observation of such behavior inspired the development of this optimization algorithm. ACO replicates the way ants promptly establish the shortest path between the nest and a food source (Bonabeau et al., 1999) [9]. Ants begin the search for food randomly around the nest. As they move, a chemical compound, named pheromone, is dropped to the ground over the path. The deposit of pheromone will create a trail and serve as an indirect way of communication between ants. Such mechanism is known as stigemergy. When a food source is found, the ant takes the food and returns back to the nest, leaving pheromone in the return path. Other ants that are randomly walking are attracted by this pheromone trail in the proportion of the amount deposited (and not evaporated yet). By following the trail, ants deposit even more pheromone. The larger the amount of pheromone, the larger the probability of the trail to be found (and followed) by other ants.

This approach has been applied to a number of combinatorial optimization problems, such as the Graph Coloring Problem, the Quadratic Assignment Problem, the Travelling Salesman Problem (Dorigo et al., 1999), the Vehicle Routing Problem (Bullnheimer et al., 1999a) (Bullnheimer et al., 1999b) (Chia-Ho et al., 2006). Methods of Ant Colony Optimization are Ant Colony System, Ranked Based Ant System, Max-Min Ant System, Ant-Q. Among these methods, this system use Ant Colony System method [2,7] combined with the Saving Heuristic.

3.1 Ant Colony System

Ant Colony System (ACS) is an extension of Ant Colony Optimization method that is a meta-heuristic approach [18]. ACS algorithm has three main parts; state transaction rule, local updating rule and global updating rule.

1. Start
2. Initialize
3. For I ^{ma} iteration do:
   A. For all ants generate a new solution
   B. Update the pheromone trails using Local updating
   C. Update the pheromone trails using Global updating
4. End

Figure1: ACS Algorithm

The ACS state transition rule provides a direct way to balance between exploration of new states and exploitation of a priori and accumulated knowledge. The local updating rule that applies to modify the pheromone trail on the routes to go to the next customers. And finally, when all ants have finished their routes, the ant that made the global updating rule to reach the least minimum cost routes.

3.1.1 State Transaction Rule

The state transaction rule also called a pseudo-random-proportional is concerned with the construction of a new solution where each ant (vehicle) decides which is the next state (customer) to move. In which, next state is chosen randomly with a probability distribution depending on η_{ij} and τ_{ij}. The best state is chosen with probability q_0 and the next state is chosen randomly with a probability distribution based on η_{ij}, τ_{ij} and γ_{ij} weighted by α, β and γ.

$$p_{ij} = \arg \max_{u \in F_k(i)} \{(\tau_{ij})^{\alpha}.(\eta_{ij})^{\beta}.(\gamma_{ij})^{\lambda}\} \ , if \ q \le q_0 \quad (1)$$

$$p_{ij} = \frac{(\tau_{ij})^{\alpha}.(\eta_{ij})^{\beta}.(\gamma_{ij})^{\lambda}}{\sum_{u \in F_k(i)} (\tau_{ij})^{\alpha}.(\eta_{ij})^{\beta}.(\gamma_{ij})^{\lambda}} \quad ,\text{if } \ q \ \phi \ q_0 \quad (2)$$

where, $\eta_{ij} = 1 / d_{ij}$ is a heuristic value, τ_{ij} denotes the pheromone concentration on the edge connecting cities i and j . The ant selects the move to expand the state taking into account the following two values:

1. the attractiveness η_{ij} of the move, as computed by some heuristic indicating the priori desirability of that move;
2. the pheromone trail level τ_{ij} of the move, indicating how useful it has been in the past to make that particular move; it represents, therefore, an a posteriori indication of the desirability of that move.

The parameters α, β and γ is the relative influence of the pheromone concentration, the heuristic value and the savings value.

3.1.2 The Saving Heuristic

This heuristic was proposed by [Clarke & Wright 1964] and improved by [Paessens 1988]. It is the basis of most of the commercial software used to solve the vehicle routing problems in the industrial applications. The objective of this heuristic is to determine whether it is better to combine the clients v_i and v_j in the same route (when the value of γ_{ij} is big) or to put them in two different routes. The Savings value of the clients v_i and v_j is calculated as follows:

$$\gamma_{ij} = d_{i0} + d_{0j} - g.d_{ij} + f.\left| d_{i0} + d_{0j} \right| \qquad (3)$$

Is the savings of combining two cities i and j on one tour as opposed to visiting them on two different tours. In which d_{ij} represents the distance between the customers i and the customer j, the index 0 corresponds to the depot, f and g represent 2 parameters of the heuristic. Where, d_{ij} is the Euclidean Distance,

$$\text{Dij} = \sqrt{(i_x - i_j)^2 + (i_y - j_y)^2} \qquad (4)$$

In which, i and j represent the coordinate values of the customers.

3.1.3 Local Updating Rule

After all the artificial ants have improved the solutions through the heuristics, the pheromone trails will be updated. This is the main feature of an ACO algorithm which assists at improving future solutions since the updated pheromone trails would reflect the ants' performance and the quality of their solutions found.

While an ant is building its solution, the pheromone level on each arc (i, j) that is visited is updated according to the local updating rule given in Equation (5)

$$\tau_{ij}^{new} = (1-\rho) + \rho \,.\, \Delta\tau_{ij} \qquad (5)$$

3.1.4 Global Updating Rule

Once all ants have built their tours then the global updating rule is applied. In the ACS method only the globally best ant is allowed to deposit pheromone in an attempt to guide the search . The global updating rule is given in the Equation (6)

$$\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \rho \,.\, \Delta\tau_{ij} \qquad (6)$$

In these local and global updating rule, ρ is the parameter of evaporation of the pheromone ($0 < \rho < 1$). If the arc (i, j) is used by an ant whose solution is accepted, the quantity of pheromone is then increased on this arc by $\Delta\tau_{ij}$ that is equal to 1/ L* with L* is the length of tours found by this ant.

3.2 Capacitated Vehicle Routing Problem

The CVRP is the basic version of the Vehicle Routing Problem (VRP), where all customers are delivery customers, the demands are known, all vehicles are identical and they belong to the same central depot. The imposed constraints are related to the capacity of the vehicles, may also be restricted in the total distance. it can travel and all customers must be served by a single route. In this problem, the objective is to find a set of delivery routes satisfying these requirements and giving minimal total travel cost. The vehicles are assumed to be homogeneous and having a certain capacity. The vehicles are assumed to be homogeneous and having a certain capacity.

The CVRP can be represented as a weighted directed graph G = (V , A) where V = { v_0 ,v_1 ,v_2 ,..., v_n } represents the set of the vertices and A = { (v_i, v_j) : i≠ j } represents the set of arcs. The vertex v_0 represents the depot and the others represent the clients. To each arc (v_i , v_j) a non-negative value d_{ij} each measured using Euclidean computations.

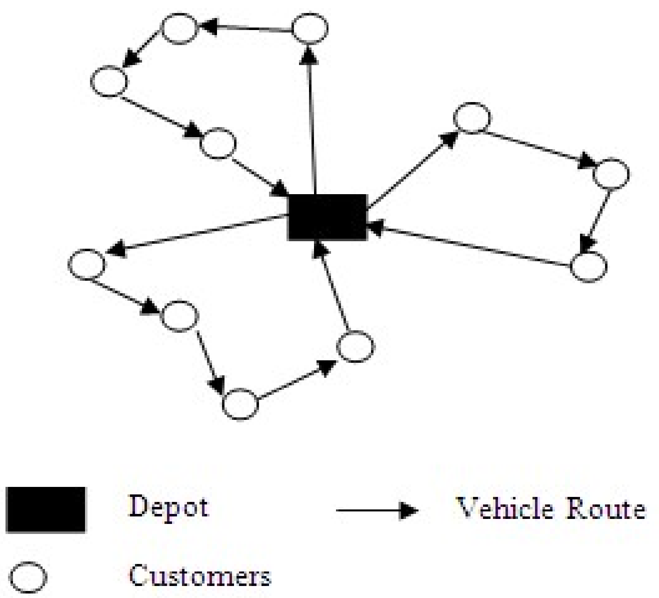

Figure 2 Example of CVRP Solution

This value corresponds to the distance between the vertex v_i and the vertex v_j in terms of cost or time between the two vertices. A demand q_i and time service δ_i (q_0=0, δ_0=0) are associated with each client (vertex) v_i. The objective is to find the minimum cost route to serve all the customers by satisfying the following constraints. (i) each customer is visited exactly once by exactly one vehicle. (ii) all vehicle routes start and end at the depot. (iii) for each vehicle route, the total demand does not exceed the vehicle capacity Q. (iv) for each vehicle route, the total route length that can travel is restricted.

3.3 Nearest Neighbor Heuristic

Nearest Neighbor Heuristic termed as Greedy Heuristic. Firstly, it needs to sort all distance between cities in ascending order. It construct a tour by repeatedly selecting the shortest distance and adding this route to the tour if it doesn't create a cycle with less than number of cities and will not allow to add the same route twice. Finally, it needs go to the start city.

4. System Design and Implementation

This section describes the system design and implementation result in different Benchmark instances (P-n20-k2 and P-n16-k8) by applying ACS algorithm with Saving heuristic. During the implementation, firstly the preprocessing steps; distance values, heuristic value and saving values are calculated. After finishing preprocessing steps, routes construction steps are calculated and finally, the system generates the minimum cost routes.

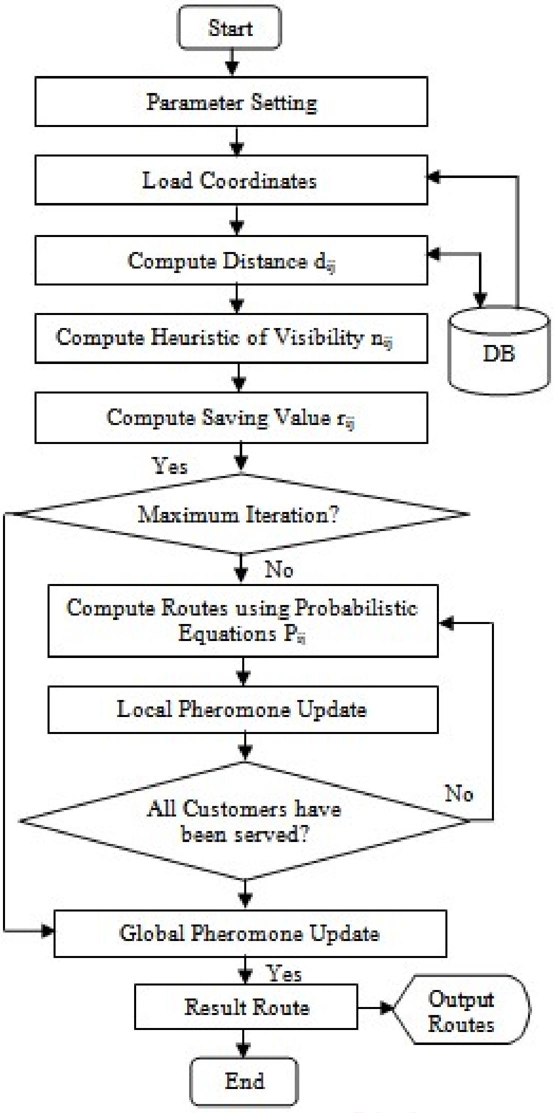

Figure 3: Flowchart of System for CVRP

In Figure 3, the processing steps of the system are calculating distance between customers using Euclidean Distance, the initial pheromone value by using Nearest Neighbor Heuristic method, the heuristic and saving value with the ACS algorithm parameters. After calculating these steps, the system start calculating the route construction step using probabilistic equation. While evaluating these steps the system updates the local pheromone update. All the customers have been served; finally, the system updates the global pheromone update and generates result of the Capacitated Vehicle Routing Problem.

4.1 Parameter Setting

From the initial investigation, we observed that the following parameter settings give a good compromise to solution quality for the system.

The number of ants m = n-1 initially placed at random on all the summits,

- $\alpha = 5, \beta = 5, \lambda=5$ with α, β and λ are three parameters that determine respectively the relative importance of the pheromone, of the distance and of Savings,
- ρ = 0.1 is the parameter of evaporation of the pheromone used in updating rules of the pheromones,
- f = g = 2 with f and g the two parameters of the heuristic of Savings,
- q_0 = 0.9 is the parameter that determines the relative importance of the exploitation versus the exploration. Before that an ant visits the next summit, q is generated randomly. If $q \le q_0$ the exploitation is then encouraged, otherwise it is the exploration process that is encouraged.

Besides, with the suggestion from Dorigo and Stutzle (2004), the initial pheromone concentration was set as $\tau_0 = m/C^{nn}$, where C^{nn} is the total length of the solution generated by the nearest-neighbor heuristic. For all the problems tested, we set the maximum iteration to 10000.

Table 1.Implementation Data

Node Coordinate Section	X Coordinate	Y Coordinate	Demand Section
1	30	40	0
2	37	52	19
3	49	49	30
4	52	64	16
5	31	62	23
6	52	33	11
7	42	41	31
…	…	…	…
20	45	35	15

In Table 1, the first column, node coordinate section represents the customers. The second and third column gives the coordinates between these customers and the last column shows the demand (need) of the customers.

Table 2 Details relative to the Instances (N: number of customers, Q: capacity of a vehicle)

Instance Name	N	Q
P-n20-k2	20	160
P-n16-k8	16	35

4.2 Construction of Vehicle Routes

The preprocessing steps of the system are calculation of distance d_{ij} between cities by using Euclidean distance formula, the initial pheromone value τ_0 by using Nearest Neighbor Heuristic method, the number of vehicle and the ACS algorithm parameters setting.

Table 3.The pre-processing Steps

Customer (i)	Customer (j)	Distance Value	Heurism Value	Saving Value
1	2	13.8924	0.0719	13.8924
1	3	21.0238	0.0476	21.0238
1	4	32.5576	0.0307	32.5576
1	5	22.0227	0.0454	22.0227
…	…	…	…	…
20	19	38.4707	0.0259	39.0423

Table 3 shows the distance value d_{ij}, the heuristic value η_{ij} and saving value γ_{ij} between customers. The Euclidean distance formula (Equation 4) is used to calculate the distance between the customers. For calculating the distance, the system used the coordinates between the customers in Table 1.

$$d_{ij} = \sqrt{(i_x - j_x)^2 + (i_y - j_y)^2}$$

$$d_{12} = \sqrt{(30-37)^2 + (40-52)^2} = 13.8924$$
$$d_{13} = \sqrt{(30-49)^2 + (40-49)^2} = 21.0238$$
$$\ldots\ldots$$
$$d_{24} = \sqrt{(37-52)^2 + (52-64)^2} = 19.2093$$

Figure 4: Calculating Distance between Customers

Attractiveness, the saving value η_{ij} is calculated by using the distances (Figure 4) for indicating how promising the choice of the next customer j is from current customer i.

$$\eta_{ij} = 1/d_{ij}$$

$$\eta_{12} = 1/d_{12} = 1/13.892 = 0.0719$$
$$\eta_{13} = 1/d_{13} = 1/21.024 = 0.0475$$
$$\ldots\ldots$$
$$\eta_{45} = 1/d_{45} = 1/21.095 = 0.0474$$

Figure 5: Calculating Heuristic Values

The calculation of saving value γ_{ij} is also depending on the distance values (Figure 4) and it calculated by applying Equation 3 as follow.

$$\gamma_{ij} = d_{i0} + d_{0j} - g.d_{ij} + f.|d_{i0} - d_{0j}|$$

$$\gamma_{12} = d_{10} + d_{02} - g.d_{12} + f|d_{10} - d_{02}|$$
$$= d_{11} + d_{12} - 2.d_{12} + 2|d_{11} - d_{12}|$$
$$= 0 + 13.892 - 2(13.892) + 2|0 - 13.892| = 13.892$$
$$\ldots\ldots$$

Figure 6: Calculating Saving Values

4.2.1 Route Construction

After being finished these steps, the system evaluates the route constructions steps using probabilistic equations (Equation 1 and Equation 2) until all the customer is served with the desired demand. In route construction step, the calculations of the three steps of ACS algorithm; State Transaction Rule, Local Updating Rule and Global Updating Rule are calculated.

Initially in the ACS, the State Transaction rule is used to construct routes by using Equation (1) and Equation (2) as follows. In this step, firstly, the m vehicles are positioned on all the routes and a quantity of initial pheromone is applied on the routes. The vehicle chooses the next customer to visit using a combination of heuristic and pheromone information (in Table 3). Each vehicle takes its departure from the depot to visit the customers. Each customer is visited once by only one vehicle, however, the depot can be visited several times. If the load stored by the vehicle exceeds the vehicle constraint capacity, the vehicle must return to the depot. When a vehicle goes back to the depot, it starts from scratch again. It initializes another route to visit other new customer. This operation is repeated over and over again, until all customers are visited. This means that a solution to CVRP has been found.

During the process of building a route, the vehicle modifies the quantity of pheromone on the chosen route by applying a local updating rule in Equation 5. Once all the vehicles are done with the building of their routing, the quantity of pheromone on the routes belonging to the best routing found is updated according to the global updating rule in Equation 6. Finally, the system generates the output result of the Capacitated Vehicle Routing Problem.

4.2.2 Implementation Result

Table 4.Implementation result for CVRP Problem

Instance Name	No. of Vehicle	Minimum Distance	No. of Route
P-n20-k2	2	219.587	2
P-n16-k8	8	450.418	8

The table 4 shows the CVRP results depends on the number of customer, the demand need by these customer and the vehicle's capacity that travel. In the first instance P-n20-k2, the vehicle with capacity Q=160 are served to (20) customers with 2 vehicles and the minimum distance is (219.587). In the second instance P-n16-k8, the vehicle with capacity Q=35 are served to 16 customers with (8) vehicles and the minimum distance is (450.418).

4.2.3 CVRP experimentations Result

Table 5.Comparing the Implementation Result with the best known published (D- The total distance travelled; V- Number of vehicle needed)

Instance Name	N	Q	Best Publish		This System	
			D	V	D	V
P-n20-k2	2	160	220	2	219.587	2
P-n16-k8	8	35	451	8	450.418	8

From Table 5 can be seen that this system shows competitive results. New best solutions are found by this approach. The solutions of problems: P-n20-k2, P-n16-k8 are improved.

Conclusion

This paper has presented solving the CVRP problem applying the Ant Colony Optimization with Saving heuristic. The CVRP consists of finding a set of at most K vehicle routes of total minimum cost, such that every route starts and ends at the depot, each customer is visited exactly once, and the sum of the demands in each vehicle route does not exceed the vehicle's capacity. We applied the system to two Benchmark (P-n20-k2 and P-n16-k8) instance of CVRP and the results obtained the minimum cost routes, minimum number of vehicles and efficient for tackling with CVRP . By applying the ACS algorithm for CVRP, we found that it avoids long convergence time by directly concentrate the search in a neighborhood of the best algorithm. So, ACS with Saving heuristic has the capability to tackle the CVRP with satisfactory solution quality and run time.

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