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Applications on Markov Chain

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ABSTRACT

Markov chain is one of the techniques used in operations research with possibilities view that managers in organizational decision making (industrial and commercial) use it. Markov processes arise in probability and statistics in one of two ways. Markov process is a tool to predict that it can make logical and accurate decisions about various aspects of management in the future.

KEYWORDS: *Markov Chain, Probability, Stochastic*

1. INTRODUCTION

Modern probability theory studies change process for which the knowledge of previous outcomes influences predictions for future experiments. In principle, when we observe a sequence of chance experiment, all of the past outcomes could influence our predictions for the next experiment. For example, this should be the case in predicting a student's grades on a sequence of exams in a course. But to allow this much generality would make it very difficult to prove general results.

In 1907, A.A. Markov began the study of an important new type of chance process. In this process, the outcome of a given experiment can affect the outcome of the next experiment.

2. SOME DEFINITIONS AND NOTATIONS

2.1 Stochastic Process

A stochastic process $\{X_t, \mathrm{t} \in \mathrm{T}\}$ is a collection of random variables.

The index t represents time and we refer X_t as the state of the stochastic process at time t. The set T is called the index set of the stochastic process. The index t is after interpreted as time and as a result, as refer X(t) as the state of the process at time.

It $\mathrm{T} = \{0, 1, 2, ...\}$, then the stochastic process is said to be discrete-time process. It T is an interval of the real line, then the stochastic process is said to be a continuous-time process. For instance $\{\mathrm{X_n}, \mathrm{n} = 0, 1, 2, ...\}$ is a discrete-time process indexed by the non-negative integers; while $\{\mathrm{X_t}, \mathrm{t} \geq 0\}$ is a continuous-time stochastic process indexed by the non-negative real numbers Let $\{\mathrm{X_n}, \mathrm{n} = 0,1,2,.....\}$ be a discrete-time stochastic process. If $X_n=i$, then the process is said to be state i at time n. A stochastic process $\{\mathrm{X_n}, \mathrm{n} = 0,1,2,.....\}$ is called a Markov chain if

$$P\{X_{n+1} = j \mid X_n = i, X_{n-1} = i_{n-1}, ..., X_1 = i, X_0 = i_0\} = P_{ij}$$

This Equation may be interpreted as stating that; for a Markov chain, the conditional distribution of any future state X_{n+1} given in the past states $X_0, X_1, ...X_{n-1}$ and the present state X_n, is independent of the past states and depends only on the present state.

The value Pij denotes the probability that the Markov chain, whenever in state i (the current state) moves next (one unit of time later) into state j and is referred to as a one-step transition probability. The transition probabilities are collected into the one-step transition probability matrix. It is denoted by P. This is,

$$P = \begin{Vmatrix} P_{00} & P_{01} & P_{02} \\ P_{10} & P_{11} & P_{12} \\ \vdots & \vdots & \vdots \\ P_{i0} & P_{i1} & P_{i2} \\ \vdots & \vdots & \vdots \end{Vmatrix}$$

Since Probabilities are non-negative and since the process must take a transition into some state, we have that

(i) $P_{ij} \geq 0, \mathrm{i}, \mathrm{j} \geq 0$

(ii) $\sum_{j=0}^{\alpha} P_{ij} = 1, i = 0, 1, 2, ...$

A state of a Markov chain is called absorbing if it is impossible to leave it. A Markov chain is absorbing if it has at least one absorbing state and if from every state it is possible to go to an absorbing state (not necessarily in one step). Markov chains have many applications as statistical models of real-world processes.

2.2 Example (beauty and intelligence)

Albert Einstein was born on born March 14th, 1879 in Germany. He died in April 18th, 1955. Marilyn Monroe was an American actress and model. She was born on 1st June of 1926 in Los Angeles, United States. She died in August 5th, 1962.

At a dinner party, Einstein and Marilyn sat next to each other. After a few flutes of champagne, she cooked in his attentive ear: "I want to have your child. With my looks and your brains, it will be a perfect child!" Einstein replied: "But what if it has my looks and your brains?"

According to this quote, we can construct Markov Model. For example:

$$\begin{array}{l} \\ \text{Albert Einstein} \\ \text{Intelligence} \\ \text{ugly} \end{array} \begin{array}{c} \text{Marilyn Monroe} \\ \begin{array}{cc} \text{Ignoramus} & \text{beauty} \end{array} \\ \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \end{array}$$

The child will be perfect child because of the combination of beauty and intelligence or the child will be ugly and ignoramus. For more details of genus model I discussed in next example.

2.3 Example (Simple Mendelian inheritance)

A certain trait is determined by a specific pair of genes, each of which may be two types, say G and g.
One individual may have:

- GG combination (dominant)
- Gg or gG, considered equivalent genetically (hybrid)
- gg combination (recessive)

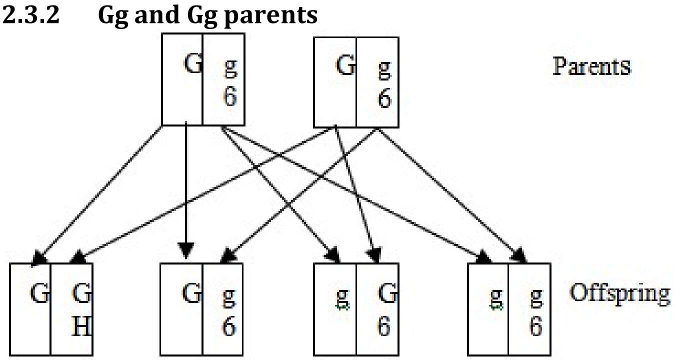
In sexual reproduction, offspring inherit one gene of the pair from each parent.

2.3.1 Basic assumption of Mendelian genetics

Genes inherited from each parent are selected at random, independently of each other. This determines probability of occurrence of each type of offspring. The offspring

- of two GG parents must be GG,
- of two gg parents must be gg,
- of one GG and one gg parent must be Gg,
- Other cases must be examined in more detail.

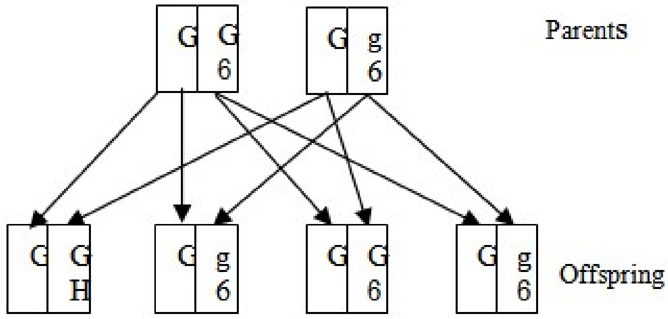
2.3.2 Gg and Gg parents



Offspring has probability

- $\frac{1}{4}$ of being GG
- $\frac{1}{2}$ of being Gg
- $\frac{1}{4}$ of being gg

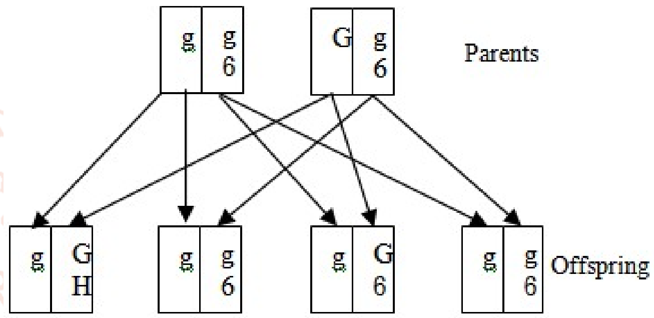
2.3.3 GG and Gg parents



Offspring has probability

- $\frac{1}{2}$ of being GG
- $\frac{1}{2}$ of being Gg

2.3.4 gg and Gg parents



Offspring has probability

- $\frac{1}{2}$ of being Gg
- $\frac{1}{2}$ of being gg

Let $p_i(n+1)$ be the probability that state S_i, $1 \le i \le r$, occurs on $(n+1)^{th}$ repetition of the experiment.

There are r ways to be in state S_i at step $n+1$:

1. Step n is S_1. Probability of getting S_1 on n^{th} step is $p_1(n)$, and Probability of having S_i after S_1 is p_{1i}. Therefore, by multiplication Principle, $P(S_i|S_1) = p_{1i}p_1(n)$.
2. We get S_2 on step n and S_i on step $(n+1)$. Then $P(S_i|S_2) = p_{2i}p_2(n)$.

r. Probability of occurrence of S_i at step $n+1$ if Sr at step n is $P(S_i|S_r) = p_{ri}p_r(n)$.

Therefore, $p_i(n+1)$ is sum of all these,

$$p_i(n+1) = P(S_i|S_1) + \cdot + P(S_i|S_r)$$
$$= p_{1i}p_1(n) + \cdot + p_{ri}p_r(n)$$

2.3.5 General case

Let $p_i(n)$ be the probability that the state S_i will occur on the n^{th} repetition of the experiment, $1 \le i \le r$.

Since one the states S_i must occur on the n^{th} repetition,
$p_1(n) + p_2(n) + \cdots + p_r(n) = 1$.

Therefore,
$p_1(n+1) = p_{11}p_1(n) + p_{21}p_2(n) + \cdots + p_{r1}p_r(n)$
....

$p_k(n + 1) = p_{1k}p_1(n) + p_{2k}p_2(n) + \cdots + p_{rk}p_r(n)$ (1)

....

$p_r(n + 1) = p_{1r}p_1(n) + p_{2r}p_2(n) + \cdots + p_{rr}p_r(n)$

In matrix form

$p(n + 1) = p(n)P, \quad n = 1, 2, 3, \ldots$ (2)

Where $p(n) = \{p_1(n), p_2(n), \ldots, p_r(n)\}$ is a (row) probability vector and $P = (p_{ij})$ is a $r \times r$ transition matrix,

$$p = \begin{pmatrix} p_{11} & p_{12} & \cdots\cdots & p_{1r} \\ p_{21} & p_{22} & \cdots\cdots & p_{2r} \\ \cdots\cdot & \cdots\cdots & \cdots\cdots & \cdots\cdot \\ p_{r1} & p_{r2} & \cdots\cdots & p_{rr} \end{pmatrix}$$

For our genetic model..

Consider a process of continued mating.

- Start with an individual of known or unknown genetic character and mate it with a hybrid.
- Assume that there is at least one offspring; choose one of them at random and mate it with a hybrid.
- Repeat this process through a number of generations.

The genetic type of the chosen offspring in successive generations can be represented by a Markov chain, with states GG, Gg and gg.

So there are 3 possible states S_1 = GG, S_2 =Gg and S_3 = gg.

So, what we have is

$[p_1(n + 1), \ldots, p_r(n + 1)] =$

$$[p_1(n), \ldots, p_r(n)] \begin{pmatrix} p_{11} & p_{12} & \cdots\cdot & p_{1r} \\ p_{21} & p_{22} & \cdots\cdot & p_{2r} \\ \cdots\cdot & \cdots\cdot & \cdots\cdot & \cdots\cdot \\ p_{r1} & p_{r2} & \cdots\cdot & p_{rr} \end{pmatrix}$$

It is easy to check that this gives the same expression as (1)

We have

GG	Gg	gg
GG	0.5	0.5
Gg	0.25	0.5
gg	0	0.5

The transition probabilities are

$$p = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.5 & 0.5 \end{pmatrix}$$

The two step transition matrix is

$$p^{(2)} = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.5 & 0.5 \end{pmatrix}\begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.5 & 0.5 \end{pmatrix} = \begin{pmatrix} 0.375 & 0.5 & 0.125 \\ 0.25 & 0.5 & 0.25 \\ 0.125 & 0.5 & 0.375 \end{pmatrix}$$

This result showed the fact that when the dominant gene and the recessive gene mixed the different types, the dominant gene reappeared in second generation and later.

3. CONCLUSION

This paper aims to present the genetic science from the mathematical point of view. To do research more from this paper, a new, well-qualified genetic one can be produced in combination with two existing genetics. In accordance with calculating or speculating the genetic probability from the mathematical aspect, it may be the logical impossibility in real world.

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