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Study on Transmission Probabilities for Some Rectangular Potential Barriers

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ABSTRACT

In this research, we apply the time independent Schroedinger equation for a particle moving in one dimensional potential barrier of finite width and height. We study the two cases which corresponds to the particle energies being respectively larger and smaller than the potential barrier. Then, we calculate transmission coefficient (T) as a function of particle energy (E) for a potential barrier by changing the barrier height (V_0) and width (L) using Propagation Matrix Method. If we keep the barrier width constant and varying the height, we see that the passing limit is shifting towards the higher energies when barrier height is increased. If we keep the barrier height constant and change the barrier width, we see significance change in oscillations.

KEYWORDS: *Rectangular Potential Barrier, Propagation Matrix Method*

INTRODUCTION

A differential equation for the wave function Ψ actually for the variation of wave function with space and time is called Schroedinger equation which describes the behavior of particles like electron, proton, neutron etc. We apply the Schroedinger equation for a particle moving in one dimensional potential barrier of finite width and height V_0. We consider a beam of particles of mass m along the x-axis from left to right on a potential barrier. According to classical physics, if a beam of particles with energy $E < V_0$ is incident on the potential barrier, it will be reflected. It cannot go through the potential barrier[1]. However, according to quantum mechanics there is finite probability that a particle with energy less than the height of potential barrier can penetrate it.

Tunneling phenomena are common at the microscopic scale; they occur within nuclei, within atoms and within solids. In nuclear physics, for instance, there are nuclei that decay into an α particle and daughter nucleus[4]. The barrier penetration effect has important applications in various branches of modern physics ranging from particle and nuclear physics to semiconductor devices. For instance, radioactive decays and charge transport in electronic devices are typical examples of the tunneling effect[5].

The Rectangular Potential Barrier

We consider a one dimensional potential barrier of finite width and height. The potential energy $V(x)$ given by Eq.(1) is called the potential barrier which has a height of V_0 and a width of L.

$$V(x)=\begin{cases} 0 & x<0 \\ V_0 & 0\le x\le L \\ 0 & x>L \end{cases} \tag{1}$$

We consider particle of mass m incident on the barrier from the left with energy E. As mentioned therein, according to classical mechanics, the particle would be reflected back if $E < V_0$ and would always be transmitted if $E > V_0$. We will show that, quantum mechanically, both reflection and transmission occur with finite probability for all values of E except in some special cases. We consider the following two cases which correspond to the particle energies being respectively larger and smaller than the potential barrier.

A. Case I ($E > V_0$)

We divide the whole space into three regions: Region I ($x < 0$), Region II ($0 < x < L$) and Region III ($x > L$). In region I and III the particle is free[2]. According to classical physics, if a beam of particles with energy $E > V_0$ approaches the potential barrier from the left, all of the particles in the beam will go over the barrier to region III.. The time independent Schroedinger equation for each region is

$$\left(\frac{d^2}{dx^2}+k_1^2\right)\psi_1(x)=0 \quad (x<0) \tag{2}$$

$$\left(\frac{d^2}{dx^2}+k_2^2\right)\psi_2(x)=0 \quad (0<x<L) \tag{3}$$

$$\left(\frac{d^2}{dx^2}+k_1^2\right)\psi_3(x)=0 \quad (x>L) \tag{4}$$

The solutions of the Schroedinger equation in the three regions are

$$\psi(x)=\begin{cases}\psi_1(x)=Ae^{ik_1x}+Be^{-ik_1x} & x\le 0\\ \psi_2(x)=Ce^{ik_2x}+De^{-ik_2x} & 0<x<L\\ \psi_3(x)=Fe^{ik_1x} & x\ge L\end{cases} \quad (5)$$

where $k_1=\sqrt{2mE/\eta^2}$ and $k_2=\sqrt{2m(E-V_0)/\eta^2}$. The potential barrier and propagation directions of the incident, reflected and transmitted waves are shown in Fig.(1) for $E > V_0$. The wave function will display an oscillatory pattern in all three regions; its amplitude reduces every time the particle enters a new region. The constants *B*, *C*, *D* and *F* can be obtained in terms of *A* from the boundary conditions. The wave functions and their first derivatives must have continuous values[3]. Solving for *F*, we obtain

$$F=4k_1k_2Ae^{-ik_1L}\left[(k_1+k_2)^2e^{-ik_2L}-(k_1-k_2)^2e^{ik_2L}\right]^{-1}$$

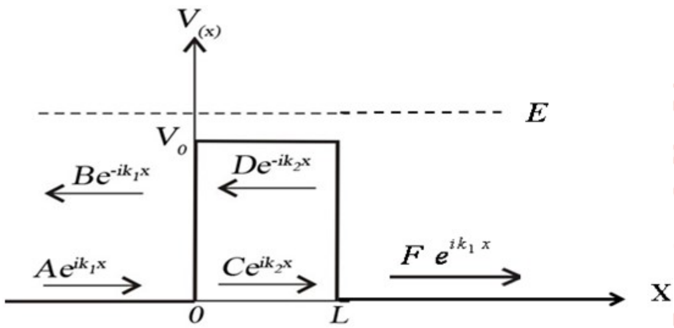

Fig.1 the potential barrier and the energy E

$$F=4\,k_1k_2A\,e^{-ik_1L}\left[4k_1k_2\cos(k_2L)-2i(k_1^2+k_2^2)\sin(k_2L)\right]^{-1} \quad (6)$$

The probability of transmission is given by the transmission coefficient *T*.

$$T=\frac{|F|^2}{|A|^2}=\left[1+\frac{1}{4}\left(\frac{k_1^2-k_2^2}{k_1k_2}\right)^2\sin^2(k_2L)\right]^{-1} \quad (E>V_0)\ (7)$$

B. Case II (E < V0)

According to classical physics, every particle that arrives at the barrier (x = 0) will be reflected back; no particle can penetrate the barrier. However, the quantum mechanical predictions differ sharply from their classical counterparts, for the wave functions is not zero beyond the barrier. In region I ($x < 0$), and region III ($x > L$), the Schroedinger equation and its solution remain the same as in case I. In region II ($0 < x < L$) the Schroedinger equation is

$$\left(\frac{d^2}{dx^2}-k_2^2\right)\psi_2(x)=0 \quad (8)$$

The solutions of the Schroedinger equation in the three regions are

$$\psi(x)=\begin{cases}\psi_1(x)=Ae^{ik_1x}+Be^{-ik_1x} & x\le 0\\ \psi_2(x)=Ce^{k_2x}+De^{-k_2x} & 0<x<L\\ \psi_3(x)=Fe^{ik_1x} & x\ge L\end{cases} \quad (9)$$

where $k_1^2=2mE/\eta^2$ and $k_2^2=2m(V_0-E)/\eta^2$. The potential barrier and propagation directions of the incident, reflected and transmitted waves are shown in Fig.(2) for $E < V_0$. The wave function has an exponential form in the forbidden region inside the barrier. But there is also an oscillator wave to the right of barrier. To find transmission coefficients $T=\frac{|F|^2}{|A|^2}$ we need only to calculate *F* in terms of *A*. The wave functions and their first derivatives must have continuous values. Solving equation for *F* / *A*, we obtain

$$\frac{F}{A}=2e^{-ik_1L}\left[2\cosh(k_2L)+i\frac{k_2^2-k_1^2}{k_1k_2}\sinh(k_2L)\right]^{-1} \quad (10)$$

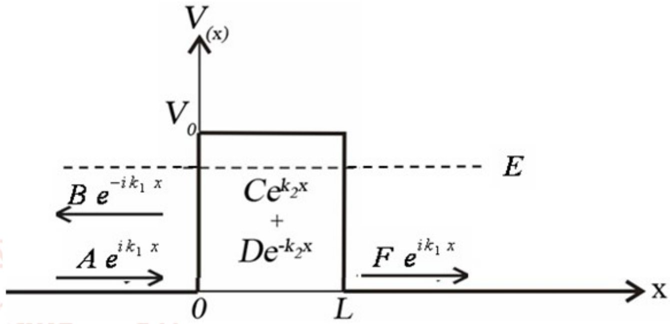

Fig (2) The potential barrier and the energy E.

$$T=\frac{|F|^2}{|A|^2}=4\left[4\cosh^2(k_2L)+\left(\frac{k_2^2-k_1^2}{k_1k_2}\right)^2\sinh^2(k_2L)\right]^{-1} \quad (11)$$

Since $\cosh^2(k_2L)=1+\sinh^2(k_2L)$, we can reduce to

$$T=\left[1+\frac{1}{4}\left(\frac{k_1^2+k_2^2}{k_1k_2}\right)^2\sinh^2(k_2L)\right]^{-1} \quad (12)$$

We note that T is finite. This means that the probability for the transmission of the particles into the region $0\le x\le L$ is not zero as expected from classical physics. This is a purely quantum mechanical effect which is due to the wave aspect of microscopic objects; it is known as the tunneling effect: quantum mechanical objects can tunnel through classically impenetrable barriers.

The Propagation Matrix Method

The Propagation Matrix

A method is needed a method for finding solutions to complicated potential structures for which analytic expressions are unmanageable. The transmission coefficient is calculated at the first potential step for a particle of energy *E* incident from the left. We then imagines the transmitted particle propagating to the next potential step, where it again has a probability of being transmitted or reflected. Associated with every potential step and free propagation region to the next potential step is a 2×2 matrix which carries wave function amplitude. The total one dimensional propagation probability for a potential consisting of a number of potential steps may be calculated by multiplying together each 2×2 matrix associated with transmission and reflection at each potential step. Therefore, the wave function coefficients for a particle traversing a one dimensional potential consisting of a number of such regions may be calculated by multiplying together the appropriate 2×2 matrices. We can solve for a particle moving in an arbitrary potential by dividing the potential into a number of potential energy steps. The following four basic parts are needed. We may use the propagation matrix method to calculate the probability of the electron emerging on the right-hand side of the barrier. The method is best approached by dividing it into small, easy-to-understand, logical parts.

The Step Propagation Matrix

We calculate the propagation matrix $\mathbf{p}_{step}$ for transmission and reflection of the wave function representing a particle of energy *E* incident on a single potential step. The potential step we consider is at position x_{j+1} in Fig.(3).

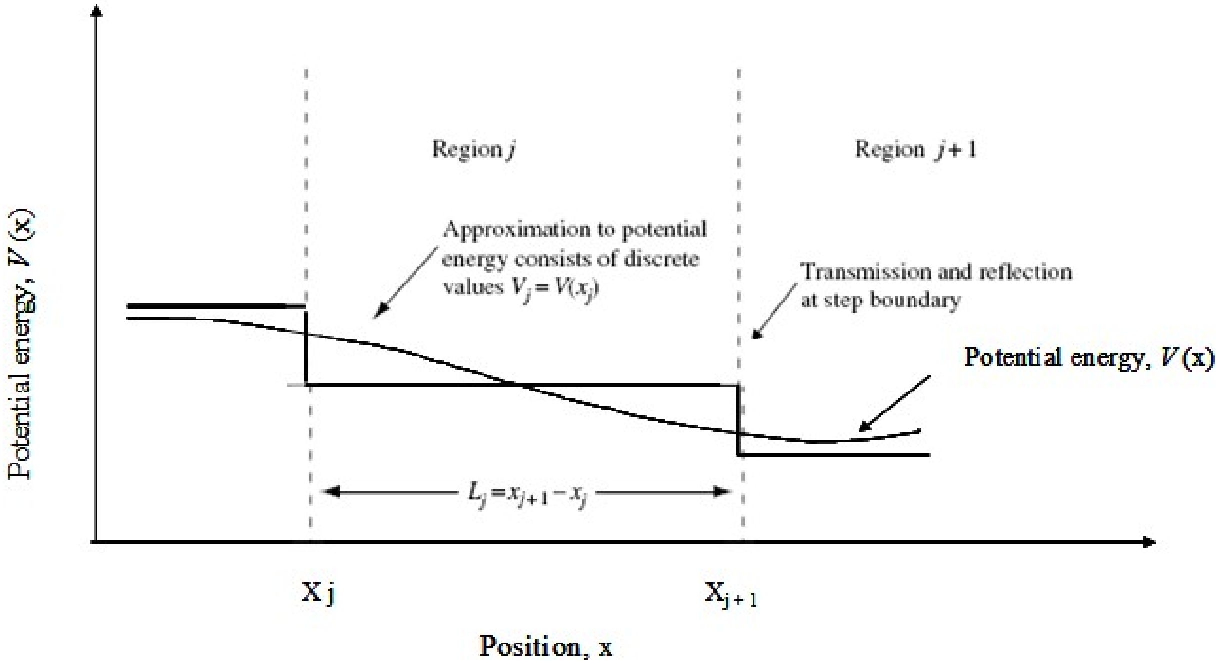

Fig.3 Approximation of a smoothly varying one dimensional potential V(x) with a series of potential steps

In this approach, the potential between position x_j and x_{j+1} in region j is approximated by a value V_j. Associated with the potential step at x_j and free propagation distance $L_j = x_{j+1} - x_j$ is a 2×2 matrix which carries all of the amplitude and phase information on the particle. Fig.(4) shows detail of the potential step at position index j +1. The coefficients *A* and *C* correspond to waves traveling left to right in region j and j +1, respectively.

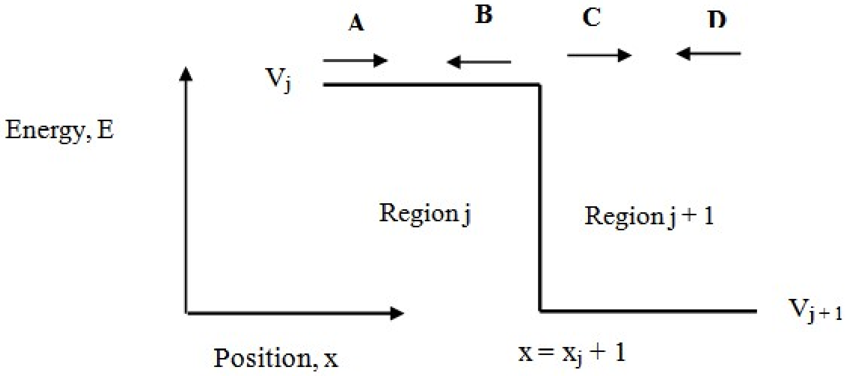

Fig.4 A one dimensional potential step. In region j the potential energy is V_j and in region j+1 the potential energy is V_{j+1}.

The transition between region 1 and region 2 occurs at position $x = x_{j+1}$. The electron (or particle) has wave vector $k_j = \frac{(2m(E - V_j))^{1/2}}{\eta}$ in region j, and the wave functions, which are solutions to the Schrödinger equation in regions j and j+1, are

$$\psi_j = A_j e^{ik_j x} + B_j e^{-ik_j x} \tag{13}$$

$$\psi_{j+1} = C_{j+1} e^{ik_{j+1} x} + D_{j+1} e^{-ik_{j+1} x} \tag{14}$$

Following the convention we have adopted in this paper, *A* and *C* are coefficients for the wave function traveling left-to-right in regions j and j+1, respectively, and *B* and *D* are the corresponding right-to-left traveling-wave coefficients. The two wave functions given by Eq.(13) and (14) are related to each other by the constraint that ψ and dΨ/dx must be continuous. This means that at the potential step that occurs at the boundary between regions j and j+1 we require

$$\psi_j\Big|_{x=x_{j+1}} = \psi_{j+1}\Big|_{x=x_{j+1}} \tag{15}$$

and $$\frac{d\psi_j}{dx}\Big|_{x=x_{j+1}} = \frac{d\psi_{j+1}}{dx}\Big|_{x=x_{j+1}} \tag{16}$$

Substituting Eq.(13) and (14) into Eq.(15) and (16) gives two equations

$$A_j\, e^{ik_j x} + B_j\, e^{-ik_j x} = C_{j+1}\, e^{ik_{j+1} x} + D_{j+1}\, e^{-ik_{j+1} x} \tag{17}$$

$$A_j\, e^{ik_j x} - B_j\, e^{-ik_j x} = \frac{k_{j+1}}{k_j}\, C_{j+1} e^{ik_{j+1} x} - \frac{k_{j+1}}{k_j}\, D_{j+1} e^{-ik_{j+1} x} \tag{18}$$

By organizing into rows and columns the terms that contain left to right traveling waves of the form e^{ikx} and right-to-left traveling waves of the form e^{-ikx}, we may write Eq.(17) and (18) for a potential step at position $x_{j+1}= 0$ as a matrix equation:

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} A_j \\ B_j \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{k_{j+1}}{k_j} & -\frac{k_{j+1}}{k_j} \end{bmatrix} \begin{bmatrix} C_{j+1} \\ D_{j+1} \end{bmatrix} \tag{19}$$

We would much prefer a simple equation of the type

$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = p_{j\,step} \begin{bmatrix} C_{j+1} \\ D_{j+1} \end{bmatrix} \tag{20}$$

where $\mathbf{p}_{jstep}$ is the 2×2 matrix describing wave propagation at a potential step. To obtain this expression, we need to eliminate the 2×2 matrix on the left-hand side of Eq.(19). We simply use from basic linear algebra that the inverse of a 2×2 matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is $A^{-1} = \frac{1}{|A|}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$ where the determinant of A is given by $|A| = a_{11}\, a_{22} - a_{12}\, a_{21}$. Hence, the inverse of $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ is $\frac{-1}{2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, so that we may write

$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \frac{k_{j+1}}{k_j} & -\frac{k_{j+1}}{k_j} \end{bmatrix} \begin{bmatrix} C_{j+1} \\ D_{j+1} \end{bmatrix} = p_{step} \begin{bmatrix} C_{j+1} \\ D_{j+1} \end{bmatrix} \tag{21}$$

where the step matrix is

$$p_{j\,step} = \frac{1}{2}\begin{bmatrix} 1+\frac{k_{j+1}}{k_j} & 1-\frac{k_{j+1}}{k_j} \\ 1-\frac{k_{j+1}}{k_j} & 1+\frac{k_{j+1}}{k_j} \end{bmatrix} \tag{22}$$

This is our result for the step potential that will be used later. We continue the development of the matrix method by considering the propagation between steps.

The Propagation between Steps

We calculate the propagation matrix $\mathbf{p}_{free}$ for propagation of the wave function between steps. The free propagation we consider is between positions x_j and x_{j+1} in Fig.(3). The distance of this free propagation is L_j. Propagation between potential steps separated by distance L_j carries phase information only so that $A_j\, e^{ik_jL_j} = C_{j+1}$ and $B_j\, e^{-ik_jL_j} = D_{j+1}$. This may be expressed in matrix form as

$$\begin{bmatrix} e^{ik_jL_j} & 0 \\ 0 & e^{-ik_jL_j} \end{bmatrix} \begin{bmatrix} A_j \\ B_j \end{bmatrix} = \begin{bmatrix} C_{j+1} \\ D_{j+1} \end{bmatrix} \tag{23}$$

Or, alternatively, $$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = P_{j\,free} \begin{bmatrix} C_{j+1} \\ D_{j+1} \end{bmatrix} \tag{24}$$

where $$p_{j\,free} = \begin{bmatrix} e^{-ik_jL_j} & 0 \\ 0 & e^{ik_jL_j} \end{bmatrix} \tag{25}$$

The Propagation Matrix P_j for the *j*-th Region

We calculate the propagation matrix for the j-th region in Fig.(3). This is achieved if we multiply $\mathbf{p}_{step}$ and $\mathbf{p}_{free}$ to obtain the propagation matrix $\mathbf{P}_j$ for the j-th region of the discretized potential. To find the combined effect of $\mathbf{p}_{free}$ and $\mathbf{p}_{jstep}$ we simply multiply the two matrices together. Hence, propagation across the complete j-th element consisting of a free propagation region and a step is

$$p_j = p_{jfree}\, p_{jstep} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \quad (26)$$

When we multiply out the matrices $\mathbf{p}_{jfree}\,\mathbf{p}_{jstep}$ given by Eq.(25) and (22), respectively, it gives us the propagation matrix for the j-th region:

$$p_j = \frac{1}{2}\begin{bmatrix} \left(1+\frac{k_{j+1}}{k_j}\right)e^{-i k_j L_j} & \left(1-\frac{k_{j+1}}{k_j}\right)e^{-i k_j L_j} \\ \left(1-\frac{k_{j+1}}{k_j}\right)e^{i k_j L_j} & \left(1+\frac{k_{j+1}}{k_j}\right)e^{i k_j L_j} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{12}^* & p_{11}^* \end{bmatrix} \quad (27)$$

Propagation through an Arbitrary Series of Step Potentials

We calculate the total propagation matrix ***P*** for the complete potential by multiplying together the propagation matrices for each region of the discretized potential. For the general case of *N* potential steps, we write down the propagation matrix for each region and multiply out to obtain the total propagation matrix,

$$P = p_1\, p_2 \ldots p_j \ldots p_N = \prod_{j=1}^{j=N} p_j \quad (28)$$

The total propagation matrix *P* satisfies continuity in Ψ and dΨ /dx between adjacent regions. Since the particle is introduced from the left, we know that $A = 1$, and if there is no reflection at the far right then $D = 0$. We may then rewrite

$$\begin{bmatrix} A \\ B \end{bmatrix} = \left(\prod_{j=1}^{j=N} p_j\right)\begin{bmatrix} C \\ D \end{bmatrix} = P\begin{bmatrix} C \\ D \end{bmatrix} \quad (29)$$

as
$$\begin{bmatrix} 1 \\ B \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}\begin{bmatrix} C \\ 0 \end{bmatrix} \quad (30)$$

In this case, because $1 = p_{11}\,C$, the transmission probability $|C|^2$ is simply

$$|C|^2 = \left|\frac{1}{p_{11}}\right|^2 \quad (31)$$

Eq.(20) is a particularly simple result. We will make use of this when we calculate the transmission probability of a particle through an essentially arbitrary one-dimensional potential.

Transmission Probability for a Rectangular Potential Barrier

Fig.(5) is a sketch of the rectangular potential barrier we will consider. The thickness of the barrier is *L*. A particle of mass m incident from the left of energy *E* has wave vector k_1. In the barrier region, the wave vector is k_2. The wave vector k_1 and k_2 are related through $k_1^2 = k_2^2 + 2\,m\,V_0/\eta^2$.

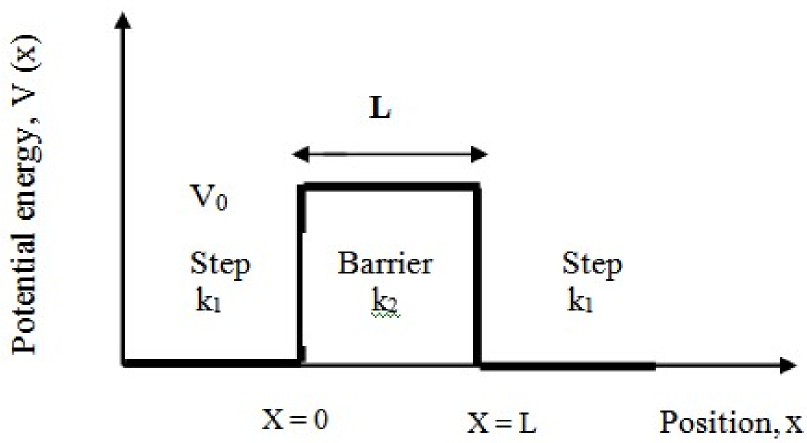

Fig.5 The potential of a one dimensional rectangular barrier of energy $\mathbf{V_0}$

A wavy particle incident on the barrier from the left with amplitude A sees a potential step-up in energy of V_0 at $x=0$, a barrier propagation region of length L, and a potential step-down at $x=L$. A particle of energy E, mass m, and charge e has wave number k_1 outside the barrier and k_2 in the barrier region $0<x<L$. We consider a particle impinging on a step change in potential between two regions in which the wave vector changes from k_1 to k_2 due to the potential step up shown in Fig.(3). The corresponding wave function changes from ψ_1 to ψ_2. Solutions of the Schroedinger equation for a step change in potential are

$$\Psi_1 = \frac{A}{\sqrt{k_1}} e^{i k_1 x} + \frac{B}{\sqrt{k_1}} e^{-i k_1 x} \tag{32}$$

$$\Psi_2 = \frac{C}{\sqrt{k_2}} e^{i k_2 x} + \frac{D}{\sqrt{k_2}} e^{-i k_2 x} \tag{33}$$

Applying the condition that the wave function is continuous at the potential step $\Psi_1|_{step} = \Psi_2|_{step}$ and that the derivative of the wave function is continuous $\left.\frac{d\Psi_1}{dx}\right|_{step} = \left.\frac{d\Psi_2}{dx}\right|_{step}$ gives

$$\frac{A}{\sqrt{k_1}} + \frac{B}{\sqrt{k_1}} = \frac{C}{\sqrt{k_2}} + \frac{D}{\sqrt{k_2}} \tag{34}$$

$$\frac{A}{\sqrt{k_1}} - \frac{B}{\sqrt{k_1}} = \frac{k_2}{k_1}\frac{C}{\sqrt{k_2}} - \frac{k_2}{k_1}\frac{D}{\sqrt{k_2}} \tag{35}$$

Rewritten in matrix form, these equations become

$$\frac{1}{\sqrt{k_1}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{\sqrt{k_2}}\begin{bmatrix} 1 & 1 \\ \frac{k_2}{k_1} & -\frac{k_2}{k_1} \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix} \tag{36}$$

To eliminate the 2 × 2 matrix on the left-hand side of this equation, we must find and multiply by its inverse matrix. The determinant of the left-hand matrix is $(-1-1)/k_1 = -2/k_1$, so the inverse of the left-hand matrix is $\frac{k_1}{2\sqrt{k_1}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. Hence, we may rewrite Eq. (36) as

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{k_1}{2}\frac{1}{\sqrt{k_1}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ \frac{k_2}{k_1} & -\frac{k_2}{k_1} \end{bmatrix}\frac{1}{\sqrt{k_2}}\begin{bmatrix} C \\ D \end{bmatrix} = \frac{1}{2\sqrt{k_1 k_2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} k_1 & k_1 \\ k_2 & -k_2 \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix}$$

Multiplying out the two square matrices gives the 2×2 matrix describing propagation at the step-up in potential

$$\begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{2\sqrt{k_1 k_2}}\begin{bmatrix} k_1+k_2 & k_1-k_2 \\ k_1-k_2 & k_1+k_2 \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix} \tag{37}$$

Since the rectangular potential barrier consists of a step up and a step down, we can make use of this symmetry and immediately calculate the 2 × 2 matrix for the step down by simply interchanging k_1 and k_2. The total propagation matrix for the rectangular potential barrier of thickness *L* consists of the step-up 2 × 2 matrix multiplied by the propagation matrix from the barrier thickness *L* multiplied by the step-down matrix. Hence, our propagation matrix become

$$\mathrm{P} = \frac{1}{2\sqrt{k_1 k_2}}\begin{bmatrix} k_1+k_2 & k_1-k_2 \\ k_1-k_2 & k_1+k_2 \end{bmatrix}\begin{bmatrix} e^{-i k_2 L} & 0 \\ 0 & e^{i k_2 L} \end{bmatrix}\frac{1}{2\sqrt{k_1 k_2}}\begin{bmatrix} k_2+k_1 & k_2-k_1 \\ k_2-k_1 & k_2+k_1 \end{bmatrix}$$

$$\mathrm{P} = \frac{1}{4k_1 k_2}\begin{bmatrix} (k_1+k_2)e^{-i k_2 L} & (k_1-k_2)e^{i k_2 L} \\ (k_1-k_2)e^{-i k_2 L} & (k_1+k_2)e^{i k_2 L} \end{bmatrix}\begin{bmatrix} k_2+k_1 & k_2-k_1 \\ k_2-k_1 & k_2+k_1 \end{bmatrix} \tag{38}$$

To find the matrix elements of *P*, we just multiply out the matrices in Eq.(38). For example p_{12} becomes

$$p_{12} = -\frac{k_2^2-k_1^2}{4k_1 k_2}\left(e^{i k_2 L} - e^{-i k_2 L}\right) \tag{39}$$

The next step we want to take is to calculate the transmission probability for a particle incident on the barrier. We already know that the transmission of a particle incident from the left is given by $\left|\frac{1}{p_{11}}\right|^2$, so that we will be interested in obtaining p_{11} from Eq.(38)

$$p_{11}=\frac{(k_2+k_1)(k_1+k_2)e^{-ik_2L}+(k_1-k_2)(k_2-k_1)e^{ik_2L}}{4k_1k_2} \tag{40}$$

$$p_{11}=\frac{(k_2^2+k_1^2+2k_1k_2)e^{-ik_2L}-(k_1^2+k_2^2-2k_1k_2)e^{ik_2L}}{4k_1k_2} \tag{41}$$

$$p_{11}=\frac{(k_2^2+k_1^2)(e^{-ik_2L}-e^{ik_2L})}{4k_1k_2}+\frac{2k_1k_2(e^{-ik_2L}+e^{ik_2L})}{4k_1k_2} \tag{42}$$

$$p_{11}=-\frac{1}{2}\frac{(k_2^2+k_1^2)(e^{ik_2L}-e^{-ik_2L})}{2k_1k_2}+\frac{1}{2}(e^{-ik_2L}+e^{ik_2L}) \tag{43}$$

$$p_{11}=-\frac{1}{2}\frac{(k_2^2+k_1^2)(e^{ik_2L}-e^{-ik_2L})}{2k_1k_2}+\frac{1}{2}(e^{-ik_2L}+e^{ik_2L}) \tag{44}$$

The Result and Discussion

We use the propagation matrix approach to calculate the transmission coefficient T of a rectangular potential barrier with width L = 1nm and height V_0 = 0.3 eV as shown in Fig.(5). Matrix technique in transmission calculations means that we express every region of barrier as a propagation matrix P. The transmission coefficient can be expressed $T=\frac{1}{|p_{11}|^2}$. Using this principle, transmission coefficient T dependence on electron energy for a rectangular potential barrier was numerically calculated in Fig.(6). We vary barrier width while keeping barrier height constant. Transmission probabilities $T(E)$ for barrier height V_0 = 0.3 eV and width L = 0.2 nm to 3.2 nm by increasing twice of initial value are shown in Fig.(7) to Fig.(11) respectively. Fig.(12) shows the transmission probability $T(E)$ for barrier height V_0 = 0.3 eV and different barrier width.

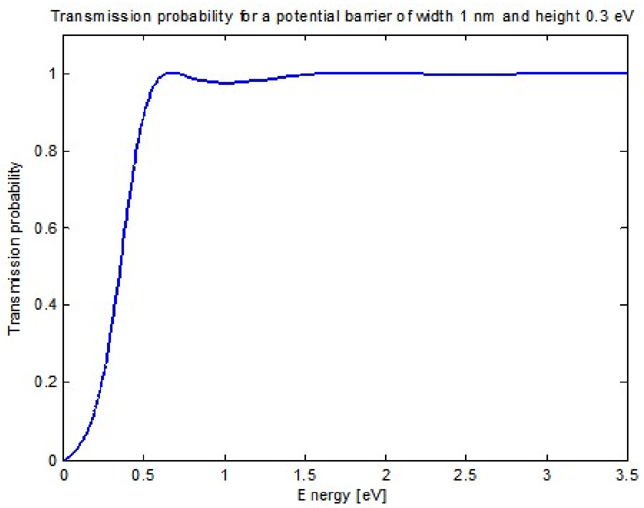

Fig.(6) The transmission coefficient T for a barrier of width L = 1 nm and height V_0 = 0.3 eV.

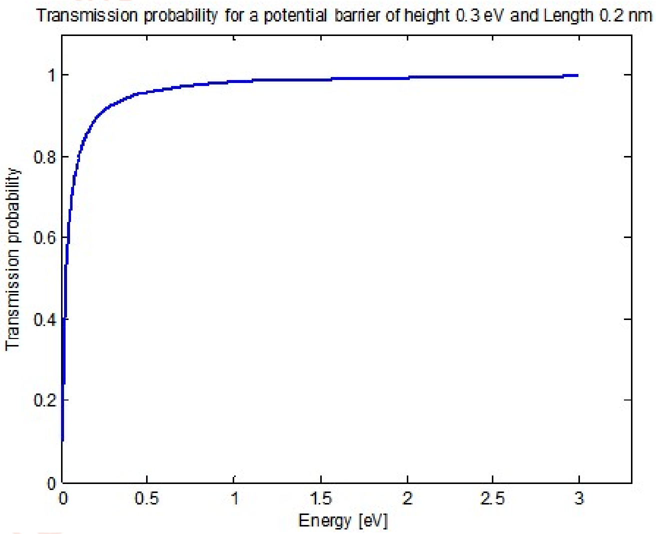

Fig.(7) Numerical calculation of transmission coefficient for a barrier of height V_0 = 0.3 eV and width L = 0.2 nm.

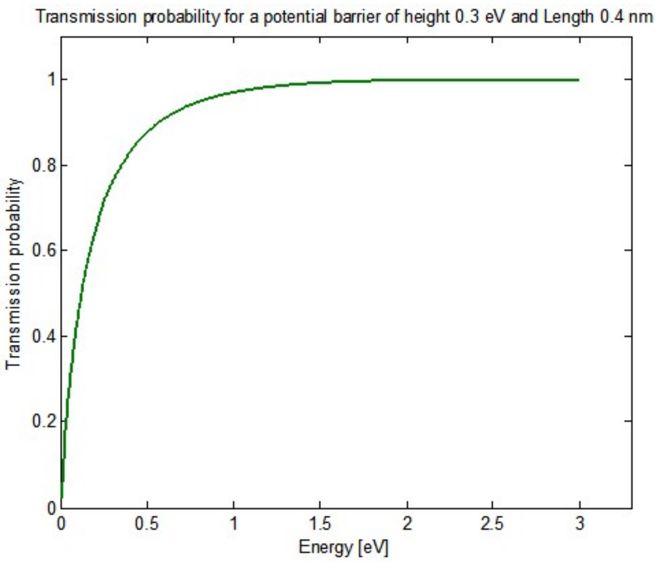

Fig.(8) Numerical calculation of transmission coefficient for a barrier of height V_0 = 0.3 eV and width L = 0.4 nm.

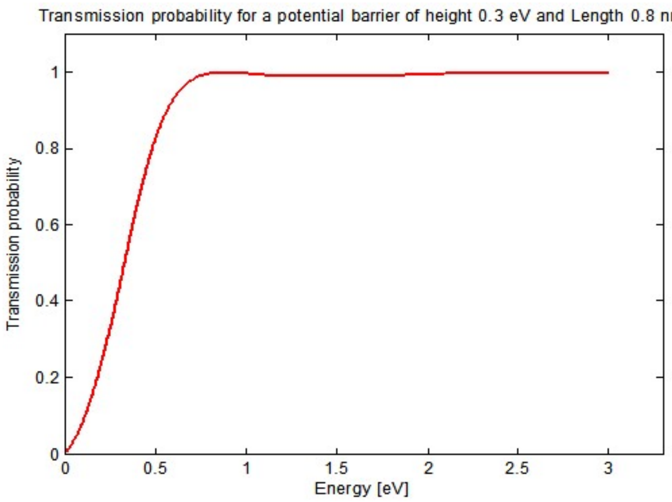

Fig.(9) Numerical calculation of transmission coefficient for a barrier of height V_0 = 0.3 eV and width L = 0.8 nm.

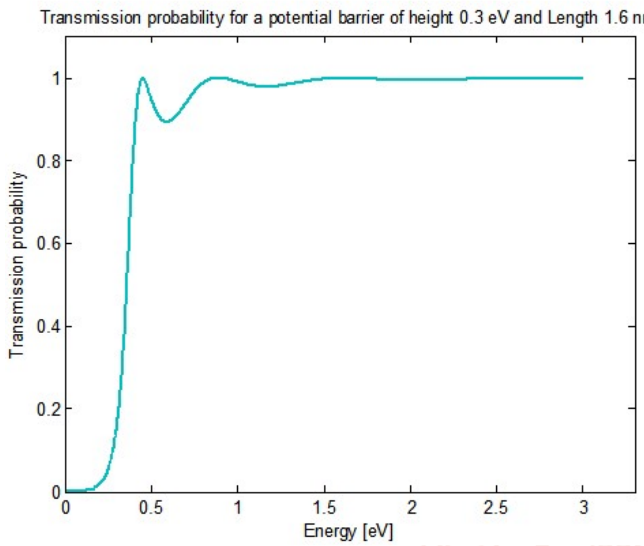

Fig.(10) Numerical calculation of transmission coefficient for a barrier of height V_0 = 0.3 eV and width L = 1.6 nm.

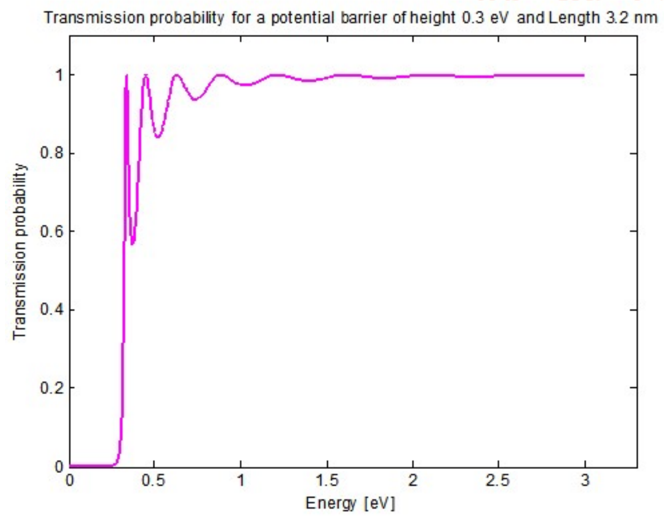

Fig.(11) Numerical calculation of transmission coefficient for a barrier of height V_0 = 0.3 eV and width L = 3.2 nm.

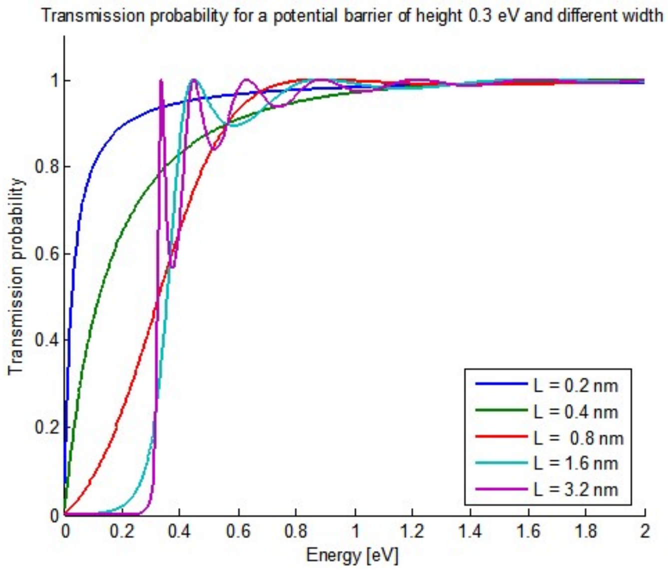

Fig.(12) Numerical calculation of transmission coefficient for a barrier of height V_0 = 0.3 eV and different width.

When a beam of particles of fixed energy is incident on a potential barrier, a certain fraction of the incident particles is transmitted while the remaining fraction is reflected. This is in contrast to classical mechanics. Classically, there must be total transmission if the energy of the incident particle is more than the height of the barrier, and total reflection if the energy of the incident particle is less than the height of the barrier. Fig.(12) shows the transmission probability $T(E)$ for barrier height V_0 = 0.3 eV and different barrier width. If we keep the barrier height constant and change the barrier width, we see significance change in oscillations. When barrier width is narrow, even electron with lower energy can pass through the barrier by quantum tunneling. The wider the barrier will be the less observable the tunneling is, which is expectation from classical approach as well.

Conclusion

We have found that the transmission coefficient (T) depends upon potential barrier height (V_0) and barrier width (L). The transmission coefficient (T) is a measure of the probability that the particle will be transmitted through the barrier. Thus, we conclude that there is finite probability of particle penetrating the barrier and appearing on the other sides. The ability of particle to penetrate the barrier when $E < V_0$ is a quantum mechanical result and is known as tunnel effect. In nuclear physics, there are nuclei that decay into an α particle and daughter nucleus. This α decay process can be explained by tunnel effect.

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