International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 3 Issue 5, August 2019 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

Minimization of Assignment Problems

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How to cite this paper*:* Mie Mie Aung | Yin Yin Cho | Khin Htay | Khin Soe Myint "Minimization of Assignment Problems" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-3 | Issue-5, August 2019, pp.1360-1362, **https://doi.org/10.31142/ijtsrd26712**

IJTSRD26712

ABSTRACT

The assignment problem is a special type of linear programming problem and it is sub-class of transportation problem. Assignment problems are defined with two sets of inputs i.e. set of resources and set of demands. Hungarian algorithm is able to solve assignment problems with precisely defined demands and resources.

Nowadays, many organizations and competition companies consider markets of their products. They use many salespersons to improve their organizations marketing. Salespersons travel form one city to another city for their markets. There are some problems in travelling which salespeople should go which city in minimum cost. So, travelling assignment problem is a main process for many business functions.

KEYWORDS: *Assignment Problem, Hungarian Algorithm, Optimization Research, Transportation Problem, Optimal Solution*

1. INTRODUCTION

The assignment problem is a special case of the transportation problem in Operation Research (OR). It arises in a variety of decision-making situations. Exactly one resource has to be assigned to each of the demands and each of the resources can be chosen at most once. We can calculate the cost function of a specific assignment as the sum of all costs in the assignment made. Appropriate resource from the set of resources available has to be assigned to each of the demands in the way that the cost of the whole assignment is minimal.

Optimality is achieved when an assignment with a minimal value of cost function is found (i.e. no other assignment with lower cost exists) [6]. There are several methods of solving the assignment problem such as

A. Enumeration method
B. Transportation model
C. Hungarian method
D. The Alternate method

One of the more famous and effective solving methods is the "Hungarian Method". This algorithm is only able to solve exactly defined assignment problems, where each demand and each resource is described with exactly defined properties. The Hungarian algorithm is an algorithm for solving a matching problem or more generally an assignment linear programming problem.

2. RELATED WORKS

In [7], the paper simplifies the algorithm of searching for the even alternating path that contains a maximal element using the minimal weighted *k*-matching theorem and intercept graph. The authors in [3] propose a new mechanism for combinatorial assignment for example, assigning schedules of courses to students based on an approximation to competitive equilibrium from equal incomes (CEEI) in which incomes are unequal but arbitrarily close together. The Hungarian algorithm was extended with fuzzy logic methods in order to be able to solve vaguely defined assignment problems without their exact formalization in [5]. This research in [7] is aimed at developing a system with which to assess the abilities of baseball players in all practical aspects of the sport and compose a team in which all the players are assigned to positions such that the collective team skill is maximized for a specific goal page title section.

In [2], the concept of assignment problem was applied to solve a problem for a Legal Firm A in Kumasi which had a difficulty in assigning nine different cases to its nine junior lawyers. Based on the data collected, Management Scientist Version 5 Software which uses Hungarian Method was used to solve the problem. Optimal assignments of the cases to the junior lawyers were obtained for the Legal Firm.

3. THEORETICAL BACKGROUND

Major Combinatorial optimization is a subset of mathematical optimization that is related to operations research, algorithm theory, and computational complexity theory.

3.1 SOME PROBLEMS IN COMBINATORIAL OPTIMIZATION

Some scheduling problems can be solved efficiently by reducing the problems to well-known combinatorial optimization problems, such as linear programs, maximum flow problems, or transportation problems [1]. In this section, we will give a brief survey of these combinatorial optimization problems.

3.1.1. LINEAR AND INTEGER PROGRAMMING

A linear program is an optimization problem of the form in equation (1).

Minimize $z(x) = c_1x_1 + \cdots + c_nx_n$ (1)
Subject to $c_1x_1 + \cdots + c_nx_n \geq b_1$

.

$c_{m1}x_1 + \cdots + c_{mn}x_n \geq b_m$

$x_i \geq 0$ for i =1, ……, n.

The most popular method for solving linear programs is the simplex algorithm. It is an iterative procedure which finds an optimal solution or detects infeasibility or unboundedness after a finite number of steps. Although the number of iteration steps may be exponential, the simplex algorithm is very efficient in practice.

3.1.2. TRANSSHIPMENT PROBLEMS

The transshipment problem is a special linear program. A transshipment problem is given by

Minimize $\sum_{(i,j)\in A} c_{ij}x_{ij}$ (2)

Subject to

$\sum_{j(j,i)\in A} x_{ij} - \sum_{j(j,i)\in A} x_{ij} = b_i$ for all $i \in$ V

$l_{ij} \leq x_{ij} \leq u_{ij}$ for all (i , j) ∈ A. .

Standard algorithms for the transshipment problem are the network simplex method and the out-of-kilter algorithm, which was developed independently by Yakovleva, Minty, and Fulkerson. Both methods have the property of calculating an integral flow if all finite bi, lij , and uij are integers.

3.1.3. THE MAXIMUM FLOW PROBLEM

For this problem, we have the linear programming formulation

Maximize u (3)

Subject to

$$\sum_{j(j,i)\in A} x_{ij} - \sum_{j(j,i)\in A} x_{ij} = \begin{cases} -v \; for \; i = s \\ v \; for \; i = s \\ 0 \; otherwise \end{cases}$$

$0 \leq x_{ij} \leq u_{ij}$ for (i, j) ⊂.

The maximum flow problem may be interpreted as a transshipment problem with exactly one supply vertex s and exactly one demand vertex t, and variable supply (demand) v which should be as large as possible.

3.1.4. BIPARTITE MATCHING PROBLEMS

Consider a **bipartite graph**, i.e. a graph G = (V, A) where the vertex set V is the union of two disjoint sets V1 and V2, and arcs A ⊆ V1×V2. A **matching** is a set M ⊆ A of arcs such that no two arcs in M have a common vertex. The problem is to find a matching M with maximal cardinality. The maximum cardinality bipartite matching problem may be reduced to a maximum flow.

3.1.5. THE ASSIGNMENT PROBLEM

The **assignment problem** is to find an assignment that is minimized. We may represent the assignment problem by the n×m-matrix C = (cij) and formulate it as a linear program: Minimize $\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij}$ (4) Subject to $\sum_{j=1}^{m} x_{ij} = 1$ i = 1, …. , n

$\sum_{i=1}^{n} x_{ij} \leq 1$ j = 1 , …. , m

$x_{ij} \in \{0,1\}$ $i = 1, \ldots, n; j = 1, \ldots, m.$

The first algorithm for the assignment problem was the Hungarian method introduced by Kuhn. It solves the assignment problem in O(n^2m) steps by exploiting its special structure.

4. SOLVING METHOD FOR ASSIGNMENT PROBLEMS

Hungarian method is used to find the proper assignment and this method is dependent upon two vital theorems, state below.

Theorem 1: If a constant is added (or subtracted) to every element of any row (or column) of the cost matrix [c_{ij}] in an assignment problem then an assignment which minimizes the total cost for the new matrix will also minimize the total cost matrix.

Theorem 2: If all $c_{ij} \geq 0$ and there exists a solution $x_{ij} = x_{ij}$ such that $\sum_i c_{ij}x_{ij} = 0$ then this solution is an optimal solution, i.e., minimizes z.

4.1 THE HUNGARIAN ALGORITHM

The Hungarian method is a combinatorial optimization algorithm that solves the assignment problem in polynomial time. This method was developed and published by Harold Kuhn, who gave the name "Hungarian method" because the algorithm was largely based on the earlier works of two Hungarian mathematicians: Denes K nig and Jan Egervary.

In order to use this method, one needs to know only the cost of making all the possible assignments. Each assignment problem has a matrix (table) associated with it. It may be noted that the assignment problem is a variation of the transportation problem with two characteristics:

A. The cost matrix is a square matrix
B. The optimum solution for the problem would be such that there would be such that there would be only one assignment in a row or column of the cost matrix.

This method works on the principle of reducing the given cost matrix to a matrix of opportunity cost. Opportunity cost here shows the relative penalties associated with assigning resource to an activity as opposed to making the best or least assignment. The following algorithm applies the above theorem to a given **n × n** cost matrix to find an optimal assignment.

Step1. Transfer the cost matrix to square matrix.
Step2. Subtract the minimal element of each row from all elements in the same row.
Step3. Subtract the minimal element of each column from all elements in the same column.
Step4. Select rows and columns across which you draw lines, in a way that all the zeros are covered and that number of lines is minimal.
Step5. Find a minimal element that is not covered by any line. Add its value to each element covered by both lines and subtract it from each element that is not covered by any line .Go back to step 4. If nothing was done in step 5, go to step 6.
Step6. Assign resources to demands starting in the top row. Assign a resource only when there is only one zero in a row. As you make an assignment delete a row and a column from which you have made it. If there is no such assignment possible, move to the next row. Stop when all assignments have been made. If you reached the bottom of the matrix, proceed to next step.

Step7. Assign resources to demands starting in the leftmost column. Assign a resource only when there is only one zero in a row. As you make an assignment delete a row and a column from which you have made it. If there is no such assignment possible, move to the next column. Stop when all assignments have been made. If no assignments were made in this step, choose a random zero value and make an assignment. Proceed to step 6.

5. EXPERIMENTAL RESULT

For a company, currently have five salespeople on road-meeting buyers. Salespeople are in Shanghai; Mumbai; Taipei; Osaka and Yangon. In order to fly to five other cities: Tokyo; Hong Kong; Beijing; Delhi and Bangkok. The problem now is which salespeople should be assign to which city to minimize the total cost of transportation. The table 1 shows the cost of airplane tickets in dollars between these cities. The problem is to find the minimum cost matching of person to city. Where should we send each salesperson in order to minimize airfare?

Table1 Cost Matrix of Assignment Problem

Dest: / Dept:	Tokyo	Hong Kong	Beijing	Delhi	Bangkok
Shanghai	155	150	99	216	229
Mumbai	188	179	110	86	147
Taipei	144	124	99	253	266
Osaka	99	135	177	224	266
Yangon	131	130	184	230	221

Hungarian algorithm can be solving this assignment problem of transportation. It can solve the best possible assignment with minimum cost. The proper results are shown in Table 2. Experiment results of this assignment problem are shown in Table 3.

Table2 Proper Results of Assignment Problem

Dest: / Dept:	Tokyo	Hong Kong	Beijing	Delhi	Bangkok
Shanghai	155	150	*99*	216	229
Mumbai	188	179	110	*86*	147
Taipei	144	*124*	99	253	266
Osaka	*99*	135	177	224	266
Yangon	131	130	184	230	*221*

Table2 Experiment Results of Assignment Problem

Shanghai	→	Beijing	99
Mumbai	→	Delhi	86
Taipei	→	Hong Kong	124
Osaka	→	Tokyo	99
Yangon	→	Bangkok	221
Total cost			*629*

6. CONCLUSION AND FURTHER EXTENSION

This paper presented the most popular method for solving assignment problem. A Hungarian Algorithm was introduced for solving minimization problems. The Hungarian Algorithm has systematic procedure and very easy to understand. From this paper, it can be concluded that this algorithm provides an optimal solution directly in few steps for the minimization assignment problem. As this algorithm consumes less time and easier to understand and apply so it can be really helpful for decision makers. This algorithm helps decision maker to minimize the costs of transportation by finding an optimum solution for transportation routes. In future work, we will apply this algorithm to various transportation-related problems.

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