Modeling of DC Motor and Choosing the Best Gains for PID Controller

Ye Htet Aung, Tin Tin Hla

Department of Electronic Engineering, Mandalay Technological University, Mandalay, Myanmar

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For PID control system, the mathematical modelling of the motor is need. DC motor drives with exact positioning system are very suitable for robot systems.

II. Related Work

Thida Aung, proposed gripper control with dc motor by using PID control. It controls the dc motor’s angle accurately for the gripper’s force control. It use parameter estimation of the dc motor for modelling purpose and use the Ziegler Nichols’ tuning method for identifying the gains for the PID control.[1]

Zin Maw Tun, proposed the modelling of the PMDC (permanent magnet DC motor). It estimate the parameters of the DC motor purposes of speed control.[2]

III. Background Theory

A. Various Types of Motors

There are two types of motor drives AC motor drives and DC motor drives. In DC drives, there are various types of motors such as Servo motor, Stepper motor and DC motor. Servo motor has proportional controller itself, so, servo motor rotate very exactly. Stepper motor is exact because of their high resolution depending on the number of stator and rotor init. But, DC motors need some type of controller for moving exact angle. These motors can be controlled by using different types of control system.

B. Control Systems for Motors

Servo motors work very accurately because they have original positioning feedback control system. So, the processor must only give the degree to be moved. Stepper Motor works with minute steps which number depending on the armature and stator windings, so it moves very accurately and do not need some type of control system. So the microprocessor needs to give only the desired angle. But, to drive the dc motor accurately, certain types of control system is needed. This research use PID control system for controlling the position of the DC motor.

C. Modeling of DC Motor

To use the control system, mathematical model of a DC motor is needed to calculate. In this research, modelling of the DC motor is approached by two methods. The first method is parameter estimating of the DC motor and the second is based on first order response (Speed) curve of the DC motor.

D. PID Control System

The proposed PID control system is used in this research to control the degree of the dc motor. P means for proportional, which amplify the error to PWM (pulse width modulation), I means for integral, which support to approach the desire angle when the error become smaller. D means for differential, which can reduce the oscillations, serve as a dumper. This control system will produce voltage to apply differential, which can reduce the oscillations, serve as a dumper.
IV. Design of the Proposed System

A. Block Diagram of the system

Figure 1. Block Diagram of the System

Ultrasonic 12 volt battery is used as power supply for the motor. The two pins of the battery are connected to motor driver’s supply and ground pins. The ground pins of battery, motor driver and Arduino microcontroller are connected together to make common ground. The output pins of the motor driver are connected to the two supply pins of the motor to give voltage and drive the motor. The supply pin of the encoder is connected to the 5-volt pin of the Arduino. The two encoder pins are connected to the pin-10 and pin-11 of the Arduino to count pulse to know the actual position of the motor by degree.

B. Flow Chart of Robotic Arm System

Firstly, define the desired position of the motor to move exactly. Then, the motor is start rotating. The actual position of the motor can be calculated by using the optical encoder. Then, calculate the error from subtracting the actual error from the desired error. Then, this error is converted to the PWM value by using the PID gains. And then, convert it to voltage to apply the motor by multiplication with 12 by 255, since the motor use 12 volt but the resolution of motor driver is 8-bit. So, its numerical value can have from 0 to 255. If the volt is positive value, the motor move clockwise direction. If the volt is negative value, the motor move anti clockwise direction. If the voltage is zero, the motor will stop from moving.

Figure 2. Flow Chart of the System

C. Modelling of the DC Motor

1. Parameters Estimation Method

Firstly, it is need to measure the resistance and inductance of the motor. By measuring, the resistance of the motor is 6(ohm) and the inductance of the motor is 0.12 (mH).

Then, it is need to calculate the electrical constant (Ke) of the motor. For the purposes of measuring the Ke, test the current, RPM (revolutions per minute), and Omega (radians per seconds) by feeding various supply voltage from 2 (volt) to 12 (volt) by using the equation as

\[ \omega = \text{RPM} \times \frac{\text{rev}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{sec}} \times \frac{360\,\text{deg}}{1\,\text{rev}} \times \frac{\pi\,\text{rad}}{180\,\text{deg}} \]  

Then, calculate the electrical constant (Ke) by using the equation

\[ \text{Ke} = \frac{V - IR}{\omega} \]  

By calculating using the equation, get the electrical constant (Ke) of the motor as 0.011.

Then it is need to calculate back EMF constant (Kb). To calculate the back EMF constant (Kb), it is need to measure
the stall current and stall voltage by pulling with loads to the motor until the rotating shaft stop with various supply while the mass and distance of the load is measured. Then calculate the stall torque by using the equation as
\[ T(\text{stall}) = \text{Mass} \times \text{Distance} \]

(3)

And then, calculating the back EMF constant (Kt) by using the equations as
\[ Kt = \frac{\text{Kt}(\text{stall})}{(\text{stall}) \times \text{gearRatio}} \]

(4)

By calculation like this, get the back EMF constant as 0.0105. Then, by using these parameters, calculate the friction coefficient by using the equation
\[ B = \frac{\text{Kt} + \omega}{\omega} \]

(5)

By using this equation, get the value of the friction coefficient as B=0.0000004592.

Then, calculate the value of time constant. To calculate the time constant, run the motor and get the speed graph. The stable speed is steady state speed. The time from 0 RPM rising point to 71 RPM is the time constant. For this system, the time constant \( \tau \) is 0.021 sec. Then, calculate the moment of inertia \( J \) by using the equation
\[ J = \frac{(\text{BR} + \text{Ke} \cdot \text{Kt})}{\omega} \]

(6)

By using this equation, get the value of moment of inertia \( J \) as 0.0000003577. Then, get the values of all parameter of the DC motor as follows.

**TABLE1 Parameters of the motor**

<table>
<thead>
<tr>
<th>No</th>
<th>Parameters</th>
<th>Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R_a</td>
<td>6</td>
<td>Ohm</td>
</tr>
<tr>
<td>2</td>
<td>L_a</td>
<td>0.12</td>
<td>mH</td>
</tr>
<tr>
<td>3</td>
<td>Ke</td>
<td>0.01</td>
<td>N.m/A</td>
</tr>
<tr>
<td>4</td>
<td>Kt</td>
<td>0.01</td>
<td>V.sec/rad</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>0.000000459</td>
<td>N.m.sec/rad</td>
</tr>
<tr>
<td>6</td>
<td>I_{eq}</td>
<td>0.0000003577</td>
<td>Kg.m²</td>
</tr>
</tbody>
</table>

The open loop transfer function of the dc motor for speed control is
\[ \frac{o(s)}{V(s)} = \frac{Kt}{(Ls + R)(s + B) + KeKt} \times \frac{12}{255} \times \frac{180}{\pi} \times 2 \]

(7)

\( Ls + R(s + B) + KeKt \) is the motor only transfer function. And convert the (0 to 255) PWM range to (0 to 12) voltage range multiplying by the values of \( \frac{12}{255} \). And convert the degree range to radium range by multiplying with \( \frac{180}{\pi} \). And, multiplying with 2 because of the using encoder type. By substituting the values of the parameters of the motor to the equation, the equation change to the following equations.

\[ \frac{o(s)}{V(s)} = s(0.00000004s^{2} + 0.0000003s + 0.00100073) \]

(8)

The closed loop block diagram of the system is

![Figure4. Closed Loop Block Diagram of the System](image)

It is need to design the transfer function of the controller. In this research, it is designed by using PID controller. Firstly, assume the controller is only included P gain.

So the closed loop transfer function of the system becomes
\[ T.F = \frac{KcG(s)}{1 + KcG(s)H(s)} \]

(9)

Assume \( H(s) = 1 \)

So, the close loop transfer function becomes
\[ T.F = \frac{0.01}{0.00000004s^{2} + 0.0000003s + 0.0010001s + 0.01Kc} \]

(10)

By using Routh-Hurwitz stability criterion, Getting the values as \( Kc = 4.2 \) and \( Pcr = 0.088 \). For PID control system, the gains are
\[ Kp = 0.22Kc \]

(11)

\[ Ki = Pcr/2 \]

(12)

\[ Kd = Pcr/3 \]

(13)

By using these equations, the values of PID gains are \( Kp=0.75 \), \( Ki=0.045 \) and \( Kd=0.03 \).

**2. Modelling of the DC Motor Based on First Order Response (Speed) Curve**

![Figure5. Speed Curve of the Motor](image)

The open loop transfer function of the motor for speed control is
\[ G(s) = \frac{c}{s + 1} \]

(14)

Where, \( c \) is the ratio of the steady state speed value of the motor to the input voltage. The steady state speed value of the motor is 120 RPM and the input voltage is 12v, so the value of \( c \) is 10. As a universal truth, the response of the dc motor will get steady state value at the value as \( \frac{s}{s+4} \). The
The motor will get the steady state value at 0.28 milliseconds. So, the value of time constant is 0.007. So, the transfer function of the motor becomes
\[ \omega(s) = \frac{10}{0.007s+} \]  
(15)

And then, the angular response of the dc motor becomes
\[ \theta(s) = \frac{10}{s(0.007s+)} \]  
(16)

Firstly, set only P gain for the system. The closed loop transfer function of the dc motor is
\[ KcG(s) \frac{1}{1 + KcG(s)H(s)} \]

So, the characteristic equation for this system, 0.007s^2 + s + 10Kc = 0. By using Routh-Hurwitz stability criterion, the value of K must be the same or greater than zero.

By sequential tuning with MATLAB, we get the value of Kc as 1. Then, substituting the value of Kc to the characteristic equation and compare to \( s^2 + 2\zeta\omega_n + \omega_n^2 \). By comparing, getting the values as \( \omega_n = 37.8, p_{cr} = \frac{2\pi}{\omega_n} = 0.166 \). For PID control system,
\[ Kp = 0.6Kc \]  
(17)
\[ Ki = 0.5Pcr \]  
(18)
\[ Kd = 0.125Pcr \]  
(19)

The values of the gains are Kp=0.6, Ki=0.083 and Kd=0.02.

D. Result and Discussions
1. Modelling Based on Parameters Estimation

2. Modelling Based on First Order Response (Speed) Curve
3. Results from motor

V. Discussion

The DC motor can't rotate exactly without any controller. If the motor is needed to move with exact speed and degree, some type of motor is needed. To get the best gain for the controller, it is needed to calculate the mathematical modelling equation for the motor. The gain for the controller can be changed according to the size of the desired angle or speed. It is better if fuzzy controller is cascaded together with the PID controller.

VI. Conclusion

Modelling of the motor needs exact data. Measure and test the six parameters of the DC motor. The values of the resistance and inductance of the motor can be measured by using RLC meter. The value for the torque of the motor can get from RC filter circuit graph. Other parameters can be measured from equation by using the values getting from supplying various voltages. And then, the gain for the PID controller will get by using Ziegler Nichols’ method. Another modelling is based on first order differential equation. These values can be get from velocity output of the voltage input. The best gains for the PID controller can also get from Ziegler Nichols’ method and Simulink with MATHLAB.

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REFERENCES


