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A Review Article on Fixed Point Theory & Its Application

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ABSTRACT
The theory of fixed point is one of the most important and powerful tools of the modern mathematics not only it is used on a daily bases in pure and applied mathematics but it is also solving a bridge between analysis and topology and provide a very fruitful are of interaction between the two. The theory of fixed points belongs to topology, a part of mathematics created at the end of the nineteenth century. The famous French mathematician H. Poincare (1854-1912) was the founder of the fixed point approach. He had deep insight into its future importance for problems of mathematical analysis and celestial mechanics and took an active part in its development.

KEYWORDS: *modern mathematics, very fruitful, celestial mechanics*

1. INTRODUCTION

Fixed point theory is a rich, interesting and exciting branch of mathematics. It is a relatively young but fully developed area of research. Study of the existence of fixed point's falls within several domains such as classical analysis, functional analysis, and operator theory, general and algebraic topology. Fixed points and fixed point theorems have always been a major theoretical tool in fields as widely apart as topology, mathematical economics, game theory, and approximation theory and initial and boundary value problems in ordinary and partial differential equations.

Moreover, recently, the usefulness of this concept for applications increased enormously by the development of accurate and efficient techniques for computing fixed points, making fixed point methods a major tool in the arsenal of mathematics.

Fixed point theory is equivalent to best approximation, variation inequality and the maximal elements in mathematical economics. The sequence of iterates of fixed point theory can be applied to find a solution of a variation inequality and the best approximation theory. The theory of fixed points is concerned with the conditions which guarantee that a map $T: X \longrightarrow X$ of a topological space X into itself. Admits one or more fixed points that are points x in X for which x = Tx. For example, a translation, i.e. the mapping $T(x) = x + a$ for a fixed a, has no fixed point, a rotation of the plane has a single fixed point (the center of rotation), the mapping $x \longrightarrow x^2$ of R into itself has two fixed points (0 and 1) and the projection $(\xi_1, \xi_2) \rightarrow \xi_1$ of R^2 into the ξ_1 - axis has infinitely many fixed points (all points of the ξ_1-axis). Existence problems of the type $(T - I)x = 0$ arise frequently in the analysis. For example, the problem of solving the equation p(z) = 0, where p is a complex polynomial, is equivalent to find a fixed point of the self maps $z - p(z)$ of C. More generally, it $D : M \rightarrow E$ is an operator acting on a subset M of a linear space E, to show that the equation $Du = 0$ [resp. $u - \lambda Du = 0$] has a solution, is equivalent to show that the map $y \rightarrow y - Dy$ [$resp. \ y \rightarrow \lambda Dy$] has a fixed point.

The earliest fixed point theorem is that of Brouwer [11], who in (1912), proved that a continuous self-mapping T of the closed unit ball R^n has at least one fixed point, that is, a point x such that $Tx = x$. Several proof of this historic result can be found in the existing literature. Another fundamental result after Brouwer's fixed point theorem was given by polish mathematicians S. Banach in (1922). Banach proved a theorem, which ensures under appropriate conditions, the existence and uniqueness of a fixed point. This result is popularly known as "Banach fixed point theorems" or the **"Banach Contraction Principle".** It states that a contraction mapping of a complete metric space into itself has a unique fixed point. It is the simplest and one of the most versatile results in fixed point theory. Being based on an iteration process, it can be implemented on a computer to find the fixed point of a contractive map, it produces approximations of any required accuracy. Due to its applications in various disciplines of mathematics and mathematical sciences, the Banach contraction principle has been extensively studied and generalized on many settings and fixed point theorems have been established.

There are large classes of mappings for which fixed point theorems have been studied. It includes contractive mappings, contraction of various order mappings, Ciric contraction, asymptotically regular, densifying etc. Apart from single mappings, pair of mappings, the sequence of mappings and family of mappings are also some of the classes of mappings that have interested mathematicians.

Review of Literature

Chu-Diaz [18] and Bryant [14] observed that it is sufficient for some iterates T^n to be a contraction in order to get a unique fixed point. Rakotch [78] and Boyd-Wong [10] have attempted to generalize the Banach contraction principle by replacing the Lipschitz constant by some real valued function whose value is less than one. But generally, in order to accommodate a variety of continuous and discontinuous functions, attempts were made to replace the contractive conditions by some general form of mapping condition (called generalized contraction). In (1977), Rhoades [80] has made a comprehensive study and compared various contractive conditions which are scattered in the literature. He also introduced some new definitions. In (1978), Rhoades enlarged his system, where a number of definitions were duplicated by Fisher work ([22], [23]). In (1980), Hegedus [32] defined the concept of generalized Banach contraction. In it appeared the diameter of a non-finite set in the inequality of definition, for the first time. This new definition was the base of many generalizations. These definitions were systematized by Park [68]. In 1992, Meszaros [54] proved the equivalence of forty-three contractive definitions and many inclusion relations. Recently, Rhoades [82] tried to obtain general results from which a lot of already known results follow as a corollary. He also obtained some new results.

On the other hand, **in (1976), Jungck [36]** generalized the Banach contraction principle by introducing a contraction condition for a pair of commuting self mapping son metric space and pointed out the potential of commuting mappings for generalizing fixed point theorems in metric spaces [37]. Jungck's results have been further generalized by considering the general type of contractive conditions on the pair of mappings by Das and Naik [19], Kasahara [44], Park ([68], [69], [70]), Ranganathan [79], Singh [95] and several others. Further generalizations have also been obtained by taking contractive type conditions for three self mappings on a metric space – one of the mappings commuting with other two by Qureshi and Awadhiya [76], Bhola and Sharma [8], Khan and Imdad [48], Fisher [26], etc.

In (1982) Sessa [88], initiated the tradition of improving commutativity conditions in metrical common fixed point theorems. While doing so Sessa introduced the notion of weak commutativity. Motivated by Sessa [88], Jungck [38] defined the concept of compatibility of two mappings, which includes weakly commuting mappings as a proper subclass. After this definition there is a multitude of compatibility like conditions such as : compatibility of type (A) (Jungck, Murthy & Cho [39], Compatibility of type (B) (Pathak and Khan [65]), compatibility of type (P) (Pathak et al. [66]), weak compatibility of type (A) (Lal, Murthy & Cho [52]), p-weak compatibility (Ume and Kim [110]) whose details can be seen in their introducing papers. In (1998), Jungck and Rhoades [40] termed a pair of mappings to be weakly compatible (or coincidently commuting) if they commute at their coincidence point. In (2001), Ahmed and Rhoades [2] producer some common fixed point theorems for compatible mappings on complete metrically convex metric spaces thereafter in (2002) Aamir and Moutawaki [1] gave some new fixed point theorems under strict contractive conditions.

In (2003) Som [108] obtained some common fixed point results for a weaker type of mappings than commuting or weakly commuting, called compatible mappings, satisfying a more general inequality condition. In (2003) Phaneendra [72] have obtained common fixed point theorems for a pair of self maps which commute at their coincidence points, called weakly compatible maps using the idea of an orbit relative to self maps.

It has been known since the paper of Kannan [42] that there exists a map possessing discontinuity in their domain but still admitting fixed points. However, in every case, the maps involved were continuous at the fixed point. Recently some authors attempted to relax continuity requirement in such results and for the work of this kind one May refer to Pant ([61], [62], [63]), Singh and Mishra [103] and Pant, Lohari and Jha [64].

In (2001) Beg [5] proved an iteration scheme for asymptotically non expansive mappings in convex metric spaces. In (2003) he [6] obtained an iteration process for non process for non linear mappings in uniformly convex linear metric spaces. He also proved in [7] fixed point set function of non expansive random mapping on metric spaces. In (2003) Fisher & Duran [25] proved some fixed point theorems for multivalued mappings or orbitally complete uniform spaces.

In (2003), Suzuki [109] generalized the result of Kanan [43]. In (2003), Popa [73] has improved the result of several authors by removing the assumption of continuity, relaxing compatibility to the weak compatibility property and replacing the completeness of the spaces with a set of four alternative conditions for four functions satisfying and implicit relations. In (2003) Proinov [75] established the Meir-Keeler type contractive conditions and the contractive definitions involving gauge functions.

The concept of **2-metric spaces** has been investigated initially by Gahler [27]. This concept was subsequently enhanced by Gahler ([28], [29]), White [112] and several others. On the other hand Iseki [33], Iseki-Sharma-Sharma [34], Khan-Fisher [47], Khan [46], Singh-Tiwari-Gupta [98] and a number of other authors have studied the aspects of fixed point theory in the setting of 2-metric space.

Khan [45], Murthy-Chang-Cho-Sharma [56], Rhoades [83], Singh-Tiwari and Gupta [98] and Naidu-Prasad [57] introduced the concepts of weakly commuting pairs of self mappings, compatible pairs of self mappings of type (A) in a 2-metric spaces, and they have proved several fixed point theorems by using the weakly commuting pairs of self mappings, compatible pairs of self mappings of type (A) in a 2-metric spaces. In (2001), Naidu [58] has proved some fixed point theorems for pairs as well as quadruples of self maps on a 2-metric space satisfying certain generalized contraction condition. In (2001) B. Singh and R.K. Sharma have proved some common fixed point theorems using the concept of compatible mappings in 2-metric spaces.

Browder and Petrysyn [13] introduced the concept of asymptotically regular maps at a point in a metric space which is defined as:

"A mapping $T : X \longrightarrow X$ of a metric space (X,d) into itself is said to be asymptotically regular at a point *x* in *X* if $Lim_{n \to \infty} d\left(T^n x, T^{n+1} x\right) = 0$".Using this concept, many authors proved a various result on a fixed point and common fixed points for such mappings in complete metric spaces, for the work of this kind one can be referred to Anderson et al [4], Guay-Singh [31], and Sharma-Yuel [93].

Rhoades et. al. [84] introduced the concept of relative asymptotic regularity for a pair of mapping on a metric space and Jungck [38] proposed the concept of compatible mappings and weakly commuting mappings. Sessa [88] and others used both cited concepts and gave many interesting results.

Singh-Virendra [101] has proved a common fixed point theorem for three weakly commuting mappings by using the concept of relative asymptotic regularity of a sequence in 2-metric spaces. Later on, using the idea of compatible mappings Singh and Sharma [105] generalized the result of Singh and Virendra [101]. B. Singh, Chauhan and Sharma [104] extends the results of Iseki and others [35] for four compatible maps and proved a common fixed point theorem using the concept of relative asymptotic regularity of sequence. Nesic [60] give a general result about fixed points for asymptotic regular mappings on complete metric spaces.

Asymptotic fixed point theory involves assumptions about the iterates of the mapping in question. It has a long history in non linear functional analysis and in fact the concept of 'asymptotic contractions' is suggested by one of the earliest version of Banach's principle attributed to Caccioppoli [15]. In (2003), Kirik [51] introduced an asymptotic version of Boyd – Wong [10] result.

In the literature of fixed point theory, many authors have extensively studied. Common fixed point theorems in Banach spaces. Bose [9] proved the same results by taking the domain a closed converse subset of a uniformly convex Banach space. Prasad [74], Sahani and Bose [85] extended the results of Bose [9], by relaxing the condition of convexity from the domain.

Recently, Som [107] obtained some fixed point theorems in uniformly convex Banach spaces, which generalize some results of Prasad [74] and Sahani – Bose [85] with respect to their mappings and inequality conditions. In (1992) Ahmad and Imdad [3] obtained some common fixed point theorem for compatible asymptotic regular mappings in Banach spaces and generalized some known results with respect to their mappings and inequality conditions. In (2003), Penot [71] proves some non expansive mappings from a closed convex subset of uniformly convex Banach spaces into itself under some asymptotic contraction assumptions.

B. Gregus [30], proved a fixed point theorem in Banach theorem in Banach space, which is called Gregus fixed point theorem and then many authors have obtained some fixed point theorems of Gregus type. (Fisher and Sessa [24]), Jungck [41], Mukherji and Verma [55] and many others. In (2001) Sushil Sharma and Bhavna Deshpande [94] have proved some fixed point theorem of Gregus type for compatible mapping in Banach Spaces.

Khan and Imdad [49] obtained some results on a fixed point of certain involution in Banach spaces. In (2003), he has introduced the concept of composite involution in Banach spaces.

Several interesting results using fixed point theory are given in **approximation theory.** During the last 130 years or so this area has attracted the attention of several mathematicians. An excellent reference is Cheney [16]. For a survey paper, one is referred to M.L. Singh [102] afterward the theorem of Brosowski [12] has been a basic important result which was the generalization of Meinardus [53].

In (1979) Singh [106] improved the results of Brosowski [12], using a fixed point theorem of Jungek [36], Sahab, Khan and Sessa [87] generalized the results of Singh [106]. Pathak, Cho and Kang (67) gave an application of Jungck's (41) fixed point theorem to best approximation theory. They extended the results of Singh (106) and Sahab et. al. (87). In (2001) Change [20] generalized the result of Sahab et. al. (87) in best approximation theory under some weaker conditions.

In (2003), Nashne [59] proved some fixed point theorems without star-shaped ness condition of domain and linearity condition of mappings in setup domain and linearity condition of mapping in the setup of normed linear space. Recently Vijayraju and Marudai [111] proved some results on common fixed compact mappings are established in the setting of normed linear space which is an extension of results of Sahab, Khan [86] and Dotson [21] as a consequence some applications of best approximations are established.

APPLICATIONS OF THE FIXED POINT THEOREMS:
Fixed point theorems have numerous applications in mathematics. Most of the theorems ensuring the existence of solutions for differential, integral, operator or other equations can be reduced to fixed point theorems. They are also used in a new area of mathematical applications eg. in mathematical economics, game theory, approximation theory, dynamic programming and solutions of non linear integral equations.

Applications to Linear Equations:
To understand the situation, we first remember that for solving such a system there are various direct methods; familiar examples in Gauss elimination method. However, an iteration or indirect method may be more efficient.

To apply the Banach contraction principle, we need a complete metric space and a contraction mapping on it.

We take the set X of all ordered n-tuples of real numbers, written

$$X = (x_1, x_2 \ldots x_n),\ y = (y_1, y_2 \ldots y_n),\ z = (z_1, z_2 \ldots z_n)$$
etc

On X we define a metric d by

$$d(x, z) = \max |x_j - z_j| \quad (1.1)$$

$X = (X, d)$ is complete

on X we define $T : X \to X$ by

$$y = Tx = Cx + b, \qquad (1.2)$$

where $C = (c_{jk})$ is a fixed real $n \times n$ matrix and $b \in x$ a fixed vector.

Writing (1.2) in components, we have

$$y_j = \sum_{k=1}^{n} C_{jk} x_{ik} + \beta_j \left(j = 1,2,\ldots n \right), \quad (1.3)$$

where $b = \left(\beta_j \right)$

Finding solutions to the system described by equations (1.3) is thus equivalent to finding the fixed points of the operator equation (1.2). In order to find a unique fixed point to T, i.e., a unique solution of equation (1.3), we apply the Banach contraction principle.

Application to Differential Equations:

We shall use the Banach contraction principle to prove the famous **Picard's theorem** which plays a vital role in the theory of ordinary differential equations. The idea of the approach is quite simple.

Let us suppose that we have the differential equation

$$\frac{dy}{dx} = F\left(x, y\right), y\left(x_0\right) = y_0$$

and we suppose that F is a continuous function of (x, y) in a domain D in the complex plane. We suppose also that F satisfies the Lipschitz condition with respect to y uniformly in x, i.e.,

$$\left| F\left(x, y_1\right) - F\left(x, y_2\right) \right| \le k \left| y_1 - y_2 \right|$$

it is not difficult to see that the above equation is equivalent to the following integral equation

$$y\left(x\right) = \int_{x_0}^{x} F\left(s, y\left(s\right)\right) ds + y_0$$

Where the unknown function is $y = y(x)$

This equation suggests the definition of a mapping which, under some conditions and in a suitable space, is a contraction mapping such that its fixed point becomes the solution to our problem.

Application to Integral Equations:

The Banach contraction principle has numerous applications to integral equations. Here we shall confine ourselves to three rather simple examples.

Theorem:

Let $K(s, t)$ be a continuous real function on the unit square $[0, 1]^2$ and let $v(s)$ be a continuous real function on [0, 1]. Then there is a unique continuous real function $y(s)$ on [0,1] such that

$$y\left(s\right) = v\left(s\right) + \int_0^s K\left(s, t\right) y\left(t\right) dt$$

1. Theorem :

Let K (s, t) and w (s, t) be continuous real functions. On the unit square $[0,1]^2$ and let v (s) be a continuous real function on [0, 1]. Suppose that

$$\left| w\left(s, t_1\right) - w\left(s, t_2\right) \right| \le N \left| t_1 - t_2 \right|$$

for all $0 \le t_1, t_2, s \le 1$. Then there is a unique continuous real function y (s) on [0,1] such that

$$y(s) = v(s) + \int_0^s K(s,t) w(t, y(t)) dt.$$

1. Theorem :

Let K (s, t, u) be a continuous function on $0 \le s, t \le 1, \ge 0$ such that

$$\left| K\left(x, t, u_1\right) - K\left(s, t, u_2\right) \right| \le N\left(s, t\right) \left| u_1 - u_2 \right|$$

where N (s, t) is a continuous function satisfying

$$\int_0^1 N\left(s,t\right) dt \le K < 1$$

for every $0 \le s \le 1$. Then for every, $v \in C\left[0,1\right]$ there is a unique function $y \in C\left[0,1\right]$ such that $y(s) = v(s) + \int_0^1 K\left(s, t, y(t)\right)$ it

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