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Reliability Analysis of Identical unit System with Arbitrary Distribution Subject to Appearance Time of the Server

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ABSTRACT

In this research paper, we analyzed the reliability measures of two units identical system who's each unit has operative and complete failure modes. Repairing of the failed unit is done by a server who grabs some time to turn up at the system. Also, the server makes alternate of the failed unit by fresh ones whenever he is not capable to fix the failed unit in a pre precise time. The expressions for different consideration of vital signs have been derived using semi-Markov process and RPT. Using arbitrary distributions to all random variable, some numerical results have been obtained to depict the behavior of MTSF, availability and profit with respect to the replacement rate.

KEYWORDS: *Parallel-Unit System, Appearance time of the server, Replacement, Maximum Repair Time and Arbitrary distributions.*

1. INTRODUCTION

Two units parallel redundant systems often found applications in reliability theory Branon(1971) Nakagawa and Osaki[1975], Lam(1997), Yadavalli(2004), Gitanjali(2014, 2019) discussed a two-unit parallel redundant system with repair maintenance. In these studies, the instantaneous entrance of repairman does not give the impression to be practical. Chander [2005], Gitanjali (2012,2014, 2016, 2017) has suggested reliability models of a standby system with arrival time and appearance time of the server.

In addition, the above studies mentioned here are mainly worried to achieve a range of reliability indices by making an assumption of exponential distributions to all random variables.

To overcome this problem various reliability models have been developed by the authors including Branson and Shah 1971; Yadavalli (2004) by considering arbitrary distributions and Erlang distributions.

Thus, in this research paper, we analyzed the reliability measures of two units identical system who's each unit has operative and complete failure modes. Repairing of the failed unit is done by a server who grabs some time to turn up at the system. Also, server makes alternate of the failed unit by fresh ones whenever he is not capable to fix the failed unit in a pre precise time. The expressions for different consideration of vital signs have been derived using semi-Markov process and RPT. Using arbitrary distributions to all random variable, some numerical results have been obtained to depict the behavior of MTSF, availability and profit with respect to the replacement rate.

NOTATIONS

E	:	Set of regenerative states
O	:	Unit is operative
p(t)/P(t)	:	pdf / cdf of the rate of repair time
$r(t)/R(t)$	:	pdf / cdf of the failure rate of the unit
$f(t)/F(t)$	:	pdf / cdf of the replacement time of the unit
$g(t)/G(t)$	:	pdf / cdf of the repair time of the unit
$w(t)/W(t)$	:	pdf / cdf of the waiting time of the server for repairing of the unit
FU_r/FU_R	:	Unit is failed and under repair / under repair continuously from previous state
FW_r/FW_R	:	Unit is failed and waiting for repair/waiting for repair continuously from previous state
FUR_p/FUR_P	:	Unit is failed and under replacement / under replacement continuously from previous state
m_{ij}	:	Contribution to mean sojourn time in state $S_i \in E$ and non-regenerative state if occurs before transition to $S_j \in E$. Mathematically, it can be written as $m_{ij} = \int_0^\infty t\, d\left(Q_{ij}(t)\right) = -q_{ij}^{*'}(0)$
μ_i	:	The mean sojourn time in state S_i which is given by $\mu_i = E(T) = \int_0^\infty P(T > t)\, dt = \sum_j m_{ij}$. where T denotes the time to system failure.
$\sim/*$	:	Symbol for Laplace Stieltjes transform / Laplace transform
Ⓢ /©	:	Symbols for Stieltjes convolution / Laplace convolution.
' (desh)	:	Symbol for derivative of the function

The possible transitions between states along with transitions rates for the system model are shown in figure 1.The states S_0, S_1, S_2 and S_3 are regenerative while the other states are non-regenerative.

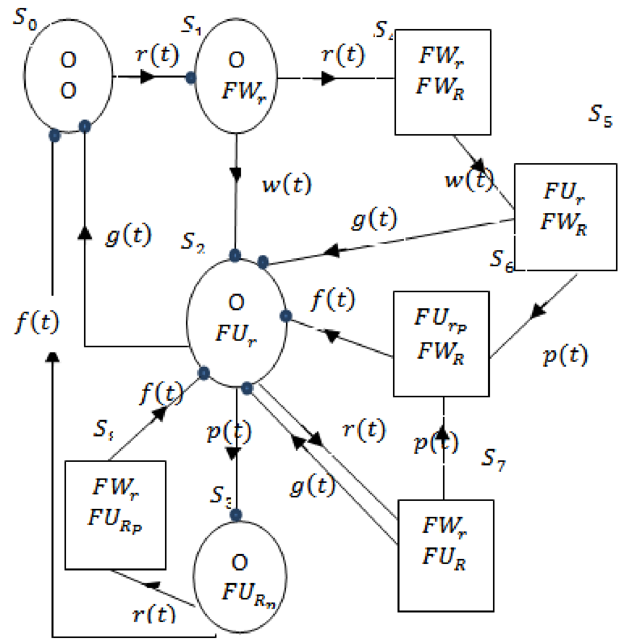

Figure1

●: Regenerative Point O : Upstate □ Failed state

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements $p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)\, dt$ as

$$p_{01} = \int_0^\infty r(t)\,dt, \quad p_{14} = \int_0^\infty r(t)\,\overline{W(t)}\,dt$$
$$p_{12} = \int_0^\infty w(t)\,\overline{R(t)}dt,\; p_{20} = \int_0^\infty g(t)\,\overline{R(t)}\,\overline{P(t)}dt,$$
$$p_{23} = \int_0^\infty p(t)\,\overline{G(t)}\,\overline{R(t)}dt,$$
$$p_{27} = \int_0^\infty r(t)\,\overline{G(t)}\,\overline{P(t)}\,dt, p_{30} = \int_0^\infty f(t)\,\overline{R(t)}\,dt,,$$
$$p_{38} = \int_0^\infty r(t)\,\overline{f(t)}\,dt, p_{45} = \int_0^\infty w(t)\,dt,$$
$$p_{56} = p_{76} = \int_0^\infty p(t)\,\overline{G(t)}dt, \quad p_{62} = \int_0^\infty f(t)\,dt,$$
$$p_{52} = p_{72} = \int_0^\infty g(t)\,\overline{P(t)}dt, \quad \ldots (1)$$

It can easily be verified that

$$p_{01} = p_{12} + p_{14} = p_{12} + p_{12.45} + p_{12.456} = p_{20} + p_{23} + p_{27} = p_{20} + p_{23} + p_{22.7} + p_{22.76} = p_{30} + p_{38} = p_{30} + p_{30} + p_{38} = p_{30} + p_{32.8} = p_{45} = p_{62} = 1$$

The mean sojourn times μ_i in state S_i is given by

$$\mu_0 = \int_0^\infty P(T > t)\,dt = \int_0^\infty \overline{r(t)}\,dt.$$
$$\mu_1 = \int_0^\infty \overline{W(t)R(t)}\,dt, \mu_2 = \int_0^\infty \overline{G(t)P(t)R(t)}\,dt$$
$$\mu_3 = \int_0^\infty \overline{F(t)R(t)}\,dt, \quad \ldots (3)$$

MEANTIME TO SYSTEM FAILURE (MTSF)

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state i to a failed state. Regarding the failed state as an absorbing state, we have the following recursive relation for $\phi_i(t)$:

$$\phi_i(t) = \sum_j' Q_{i,jj}(t)\,\boxed{S}\,\varphi_j(t) + \sum_k' Q_{i,jk}(t) \quad \ldots (4)$$

Taking $L.S.T$ of relations (4) and solving for $\tilde{\phi}_0(s)$, we get

$$MSTF(T_0) = \lim_{s\to 0}\frac{1-\tilde{\phi}_0(s)}{s} = \frac{\mu_0 + \mu_1 + p_{12}\mu_{2+}p_{12}p_{23}\mu_3}{1 - p_{12}p_{20} - p_{12}p_{23}p_{30}} = \frac{N_1}{D_1}$$

AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is in upstate at instant t given that the system entered regenerative state i at $t = 0$. The recursive relation $A_i(t)$ is given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \quad \ldots(6)$$

where

$M_0(t) = e^{-2\lambda t}$, $M_1(t) = e^{-\lambda t}\overline{W}(t)$, $M_2(t) = e^{-(\lambda+\alpha_0)t}$, $M_3(t) = e^{-\lambda t}\bar{F}(t)$

Taking $L.T.$ of relation (6) and solving for $A_0^*(s)$, we get steady-state availability as

$$A_0 = \lim_{s\to 0} s\,A_0^*(s) = \frac{(M_0^*(0) + M_1^*(0))(p_{20} + p_{23}p_{30}) + (M_2^*(0) + p_{23}M_3^*(0))}{(\mu_0 - \mu_1)(p_{20} + p_{23}p_{30}) + (\mu_2 + p_{23}\mu_3)} = \frac{N_2}{D_2} \ldots (7)$$

BUSY PERIOD ANALYSIS DUE TO REPAIR

Let $B_i^1(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered the regenerative state i at $t = 0$. The recursive relations for $B_i^1(t)$ are given as

$$B_i^1(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^1(t) \quad \ldots(8)$$

where

$$W_2(t) = e^{-(\lambda+\alpha_0)t}\bar{G}(t) + \left(\lambda e^{-\lambda t} © 1 © e^{-\alpha_0 t}\right)\bar{G}(t) \quad \ldots (9)$$

Taking $L.T.$ of relations (9) and solving for $B_0^{1*}(s)$, we get in the long run the time for which the system is under repair is given by

$$B_0^1 = \lim_{s\to 0} s\,B_0^{1*}(s) = \frac{N_3}{D_2} \quad \ldots(10)$$

Where $N_3 = P_{01}w_2^*(0)$ and D_2 is already specified.

BUSY PERIOD ANALYSIS DUE TO REPLACEMENT

Let $B_i^2(t)$ be the probability that the server is busy in replacing the unit at an instant 't' given that the system entered the regenerative state i at $t = 0$. The recursive relation for $B_i^2(t)$ are given by:

$$B_i^2(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^2(t) \quad \ldots (11)$$

Where

$$W_3(t) = e^{-\lambda t}\bar{F}(t) + \left(\lambda e^{-\lambda t} © 1\right)\bar{F}(t) \quad \ldots(12)$$

Taking $L.T.$ of relations (11) and solving for $B_0^{2*}(s)$, we get the time for which the system is under replacement is given by

$$B_0^2 = \lim_{s\to 0} s\,B_0^{2*}(s) = \frac{N_4}{D_2} \quad \ldots(13)$$

Where $P_{23}w_3^*(0)$ and D_2 is already specified.

EXPECTED NUMBER OF VISITS BY THE SERVER

Let $N_i(t)$ be the expected number of visits by the server in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. The recursive relation for $N_i(t)$ are given by

$$N_i(t) = \sum_j Q_{i,j}(t)\,\boxed{S}\left[\delta_j + N_j(t)\right] \quad \ldots (14)$$

Taking $L.S.T.$ of relations (14) and solving for $\bar{N_0}(s)$, we get the expected number of visits per unit time as

$$N_0 = \lim_{s\to 0} s\,\bar{N_0}(s) = \frac{N_5}{D_2} \quad \ldots(15)$$

Where $N_5 = P_{20} + P_{23}P_{30}$ and D_2 is already specified.

EXPECTED NUMBER OF REPLACEMENTS OF THE UNIT

Let $R_i(t)$ be the expected number of replacements by the unit in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. The recursive relation for $R_i(t)$ are given by:

$$R_i(t) = \sum_j Q_{i,j}(t)\boxed{S}[\delta_j + R_j(t)] \quad \ldots(16)$$

Taking $L.S.T.$ of relations (16) and solving for $\bar{R_0}(s)$, we get the expected number of replacements of the unit per unit time as

$$R_0 = \lim_{s\to 0} s\,\bar{R_0}(s) = \frac{N_6}{D_2} \quad \ldots(17)$$

Where $N_6 = P_{12.456}(P_{20} + P_{23}) - P_{23}P_{30}$ and D_2 is already specified.

COST-BENEFIT ANALYSIS

Profit incurred to the system model in steady state is given by:

$$P = K_1A_0 - K_2B_0^1 - K_3B_0^2 - K_4R_0 - K_5N_0$$

Where

K_1 = Revenue per unit uptime of the system

K_2 = Cost per unit time for which server is busy due to repair

K_3 = Cost per unit time for which server is busy due to replacement

K_4 = Cost per unit time replacement of the unit

K_5 = Cost per unit visits by the server

PARTICULAR CASE

Let us consider $g(t) = \theta e^{-\theta t}$, $f(t) = \beta e^{-\beta t}$

By using the non-zero element p_{ij}, we obtain the following results:

$MTSF(T_0) = \frac{N_1}{D_1}$,

Availability $(A_0) = \frac{N_2}{D_2}$,

Busy Period for repair $(B_0^1) = \frac{N_3}{D_2}$,

Busy period for replacement $(B_0^2) = \frac{N_4}{D_2}$,

Expected number of visits $(N_0) = \frac{N_5}{D_2}$,

Expected number of replacement $(R_0) = \frac{N_6}{D_2}$

Where $N_1 = \frac{1}{2\lambda} + \frac{1}{(\gamma+\lambda)} + \frac{\gamma(\alpha_0+\beta+\lambda)}{(\gamma+\lambda)(\theta+\lambda+\alpha_0)(\beta+\lambda)}$,

$$D_1 - 1 - \frac{\gamma}{(\gamma+\lambda)(\theta+\lambda+\alpha_0)}(\theta + \frac{\alpha_0\beta}{(\beta+\lambda)})$$

$$N_2 = \frac{1}{\theta+\lambda+\alpha_0}\left[1 + \frac{\theta}{2\lambda} + \frac{\theta}{(\gamma+\lambda)} + \frac{\alpha_0}{(\beta+\lambda)}\left(1 + \frac{\beta}{2\lambda} + \frac{1}{(\gamma+\lambda)}\right)\right],$$

$$N_3 = \frac{1}{\theta+\lambda+\alpha_0}\left[1 + \lambda - \frac{\alpha_0\lambda}{\beta+\lambda}\right],$$

$$N_4 = \frac{1}{\theta+\lambda+\alpha_0}\left[\frac{\alpha_0(1+\lambda)}{\beta+\lambda}\right], \qquad N_5 = \frac{\alpha_0\beta+\theta(\beta+\lambda)}{\theta+\lambda+\alpha_0(\beta+\lambda)}$$

$$N_6 = \frac{\alpha_0}{\theta+\lambda+\alpha_0}, \quad and \quad D_2 = \frac{1}{\theta+\lambda+\alpha_0}\left[1 + \frac{\lambda\alpha_0}{\theta+\alpha_0}\left(\frac{1}{\alpha_0} + \frac{1}{\beta}\right) + \frac{\alpha_0}{2\lambda} + \frac{\theta}{2\lambda}\right]$$

CONCLUSION

The numerical manners of reliability attribute MTSF, availability and profit with respect to replacement rate(β) has been shown in Tables 1,2 and 3 respectively. The values of these performance measures go on increasing with the increase of replacement rate (β), arrival rate (γ) of the server and repair rate(θ) of the unit. Thus two units identical system where the server takes appearance time and the system is analyzed by arbitrary distributions are beneficial by paying least amount for alternate unit whenever repair time taken by the server is too long and Increasing entrance rate of the server.

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β ↓	$\theta = 2.1$, $\alpha_0 = 5$, $\gamma = 3$, $\lambda = .01$	$\theta = 4.2$, $\alpha_0 = 5$, $\gamma = 3$, $\lambda = .01$	$\theta = 2.1$, $\alpha_0 = 10$, $\gamma = 3$, $\lambda = .01$	$\theta = 2.1$, $\alpha_0 = 5$, $\gamma = 6$, $\lambda = .01$	$\theta = 2.1$, $\alpha_0 = 5$, $\gamma = 3$, $\lambda = .02$
5	8266	9217	8739	11287	2100
10	9318	10211	10165	13365	2364
15	9732	10593	10753	14241	2467
20	9954	10796	11073	14725	2523
25	10091	10921	11275	15031	2557
30	10186	11006	11413	15242	2581
35	10254	11067	11515	15397	2598
40	10306	11114	11592	15515	2611
45	10346	11151	11652	15609	2621
50	10379	11180	11701	15684	2629

Table 1: MTSF Vs. Replacement Rate (β)

β ↓	$\theta = 2.1$, $\alpha_0 = 5$, $\gamma = 3$, $\lambda = .01$	$\theta = 4.2$, $\alpha_0 = 5$, $\gamma = 3$, $\lambda = .01$	$\theta = 2.1$, $\alpha_0 = 10$, $\gamma = 3$, $\lambda = .01$	$\theta = 2.1$, $\alpha_0 = 5$, $\gamma = 3$, $\lambda = .02$	$\theta = 2.1$, $\alpha_0 = 5$, $\gamma = 6$, $\lambda = .01$
5	0.995966	0.996385	0.996186	0.992059	0.998510
10	0.996233	0.996740	0.996726	0.992961	0.998746
15	0.996450	0.996859	0.996906	0.993261	0.998824
20	0.996559	0.996918	0.996996	0.993410	0.998863
25	0.996624	0.996953	0.997049	0.993500	0.998887
30	0.996667	0.996977	0.997085	0.993560	0.998902
35	0.996698	0.996994	0.997111	0.993602	0.998913
40	0.996722	0.997006	0.997130	0.993634	0.998922
45	0.996740	0.997016	0.997145	0.993659	0.998928
50	0.996754	0.997024	0.997157	0.993679	0.998933

Table 2: Availability Vs. Replacement Rate (β)

β ↓	K_1=5000, K_3=100, K_5=150					
	θ=2.1, α_0=5, γ=3, λ=.01 K_2=600, K_4=450	θ=4.2, α_0=5, γ =3, λ=.01, K_2=600, K_4=450	θ=2.1, α_0=10, γ =3, λ=.01, K_2=600, K_4=450	θ=2.1, α_0=5, γ =3, λ=.02, K_2=600, K_4=450	θ=2.1, α_0=5, γ =6, λ=.01, K_2=600, K_4=450	θ=2.1, α_0=5, γ =3, λ=.01, K_2=450, K_4=600
5	4966.742	4971.471	4967.988	4934.516	4979.391	4965.078
10	4970.119	4973.915	4971.487	4941.076	4981.660	4968.451
15	4971.244	4974.729	4972.653	4943.261	4982.415	4969.575
20	4971.807	4975.136	4973.237	4944.353	4982.793	4970.137
25	4972.145	4975.380	4973.586	4945.008	4983.019	4970.475
30	4972.370	4975.543	4973.820	4945.445	4983.170	4970.699
35	4972.531	4975.659	4973.986	4945.757	4983.277	4970.860
40	4972.651	4975.746	4974.111	4945.991	4983.358	4970.980
45	4972.745	4975.814	4974.208	4946.173	4983.421	4971.074
50	4972.820	4975.868	4974.286	4946.319	4983.471	4971.149

Table 3: Profit Vs. Replacement Rate (β)