As the result of this article I found that the relationship of differential equations with economics has been mostly closed and expanded, and solution of many issues in economics depends on formation and solving of differential equations.

**INTRODUCTION**

Differential equations are applied to most of the economic functions. These equations are used in determining the conditions for the changeable stability for the balance of economic breakdowns and the tracking of time paths under various macroeconomic conditions. If growth rates are a supposed function, economists are able to determine the sightly function by using differential equations. Also, if a point is calculated, the demand function can be calculated, differential equations are used for calculating investment functions, as well as calculation of total cost and total cost functions from final cost functions and final income. In this research paper, the linear equation of the first order in the market equilibrium is investigated.

**OBJECTIVES OF THIS RESEARCH**

The main purpose of this research paper is to obtain market equilibrium by a linear first order differential equation.
tasks in innovative ways to solve differential equations were by primary methods of addition, subtraction, multiplication, division, and analysis, with an infinite number of factors depending on the functions, differential calculus, and integral. There were several particular methods accidentally designed, such as the decoding of variables and the use of factors of integral before the end of the seventeenth century.

In the 18th century, more principled procedures were introduced by Euler, Lagrange and Laplace. It quickly appeared that relatively less differential equations could be solved by primary methods. Mathematicians have gradually found that attempting to discover methods for solving all differential equations is vain. Instead, the question is raised in this section that does a differential equation have an answer or not, and if it does, then the attempt is to find the properties of the answer is more effective than the differential equation. In this case, mathematicians found differential equations as new sources of functions.

The massive progress of physics in the last two centuries should be borne in mind by the theory of differential equations. This theory, which consists of ordinary and partial differential equations, is a branch of mathematical analysis, which is closely related to physics. Differential equations in the mid-18th century formed the basis of theoretical physics. Equations in the middle of the century were the most attractive part of this theory. Dalambert introduced vibrating string and got the first partial differential equations, which is then modeled as the main subject of his work. (porkazimi, 2009)

Then Euler was interested in this equation and he subtilized the unique conditions of the answer. These conditions were resulted the physical concept of the theorem. Euler’s articles found the way to find solutions to boundary value problems in the form of trigonometric sequences. This method was later used by Bernoulli in vibrating string, and introduced a wider range of permissible functions in setting the initial conditions. The methods of Dalambert, Euler, and Bernoulli lead to a long discussion of the effectiveness of the techniques, and then Lagrange and Laplace entered the debate.

Euler in his works has addressed other partial differential equations with complete differential equations, he developed the theory of partial differential equations and then he was identified as the innovator of partial differential equations theory. The trigonometric sequences method was used to obtain answers to quaternary issues by Fourier, which “completed it in the heat conduction equation to the application”.

Fourier presented several expanded articles on this issue, among which heat-analytical theory is of particular importance. The use of trigonometric sequences in the heat conduction theory created a new topic of mathematical analysis, called the theory of Fourier sequences.

Partial differential equations came to the other part of physics in which Laplace obtained the potential for interaction between two masses in the equation which is now called the Laplace equation. Because of the electrostatic phenomena and the theory of magnetism also appear, at the beginning of the 19th century, the general theory of potentials was innovated by Green Goss and Pawson.

These results led to the construction of partial differential equations that have a direct relationship with the boundary value problems. This topic was at the height of the nineteenth century and the beginning of the twentieth century, with the main directors of Kochi, Riemann, Poincare, Piker, Holmgen, Had Amar, Garza, and Wizard. The geometric nature of the answers to the partial differential equations, in particular the characteristic theory, opened up another way in this theory.

In the last few decades, new methods have been devised that stand by functional analysis, distribution theory, and generalized functions, and have been greatly works related to these issues accomplished by Schauder and Leray. This vital and useful theory is now the area of the most important mathematical research, and every new attractive matter emerges and grow in it.

Recently, the works and writings of some famous scientists and professors are also seen in the field of differential equations in different parts, but these investigations and writings are not sufficient due to the greatness of the discussion of differential equations, especially in the application of ordinary differential equations. First, there has not been much research in economics now.

I hope that this article, which has been written in spite of a series of constraints, can introduce the first-order ordinary differential equations in economic sand reflects the practical importance of the first-order linear differential equations good as for dear instructors and students.

**Stable and unstable balance detection:**

One of the conditions for stable balance is its steady state. It means, if for any reason the deviation from balance is achieved, effective forces to operations will lead the market, the amount and the price to balance. For example, in a condition that supply has positive slope and demand has negative slope (typical slope of supply and demand), if we reduce the price for any reason that there is a tendency to return to the initial state, then the balance is stable.

The balance is unstable when moving from the balance point leads to the entry of forces in the market that diverges us from the balance point. If the price is dropped for any reason and with increasing supply, in order to sell the product, due to the increase in supply, the decline in prices should continue to decline.

So, momentarily, we get away from the balance, this is an unstable balance. If the supply and demand curves are consistent, the balance is low or neutral, it means, if we lose the state of balance, there is no factor that causes a distant or near-balance.

**Use of first-order linear differential equation in market balance**

Demand function $Q_d = c + hp$ and supply function $Q_s = g + hp$. Suppose to consider a function of time.

We suppose that the rate of price changes in the market $dp/dt$ is being positive linear function of demand $Q_d - Q_s$, the price of balance equals to $p = c - g \over {h-b}$.
as \( \frac{dp}{dt} = m(Q_d - Q_s) \) the stability conditions of price change in the market (the conditions that when tend to infinity, \( p(t) \) is tended to \( p \)) can be calculated as follow:

\[
\frac{dp}{dt} = m(Q_d - Q_s)
\]
\[
\frac{dp}{dt} = m[(c + bp) - (g + hp)] = m(c + bp - g - hp)
\]

The above equation has become the form of a first-order linear differential equation, as we are properly familiar with the methods of its solving from differential equations, so it is calculated as follow:

\[
\frac{dp}{dt} = m(h - b)p - m(c - g)
\]
\[
\frac{dp}{dt} + m(h - b)p = m(c - g)
\]
\[
\nu = m(h - b)
\]
\[
z = m(c - g)
\]

We know the general solution form of the first-order linear differential equation that:

\[
p(t) = e^{-\int \nu dt} \left[ A + \int e^{\int \nu dt} dz \right] = e^{-\int \nu dt} \left[ A + \int e^{\nu t} dt \right] = e^{-\int \nu dt} \left[ A + \frac{z}{\nu} \right]
\]

if \( t = 0 \), the equation take the following shape:

\[
p(t) = \left[ p(0) + \frac{z}{\nu} \right] e^{-\int \nu dt} + \frac{z}{\nu} = 0
\]
\[
\Rightarrow p(0) = A + \frac{z}{\nu} \Rightarrow A = p(0) - \frac{z}{\nu}
\]

By laying down

\[
\nu = m(h - b)
\]
\[
z = m(c - g)
\]

The following link is achieved:

\[
\lim_{p \to \infty} p(t) = p(e) = \frac{c - g}{h - b} \Rightarrow p(t) = \left[ p(0) - \frac{c - g}{h - b} \right] e^{-\int \nu dt} + \frac{c - g}{h - b}
\]

because \( m \) is an constant amount and is bigger than zero, when \( t \) approaches to infinity, only when \( h - b > 0 \), the first right limit approaches to zero, so \( p(t) \) approaches to \( p \) for normal states where the demand has a negative slope \( b < 0 \) and the supply has a positive slope \( h > 0 \), the changeable stable conditions is obtained.

The markets that the slope of the demand functions are positive or the slope of supply functions are negative, when \( h > 0 \) are changeably stable as well. (porkazimi, 2009)

Example 1: Whenever the demand equation is\( D = 80 - 4p \) and the supply equation is\( S = 2p - 10 \) find the Market Balance and suppose that \( p_0 = 18 \) and \( q_0 = 8 \) find wether the balance is stable or not?

Solution: as we know, the balance point is equal to

Supply equation = demand equation

\[
2p - 10 = 80 - 4p
\]
\[
q_e = 20 \quad p_e = 15,
\]
and based on

\[
\frac{dp}{dt} + m(h - b)p = m(c - g)
\]
\[
\nu = m(2 + 4)
\]
\[
z = m(80 + 10)
\]
\[
p(t) = Ae^{-6mt} + 15 p(t) = Ae^{-6mt} + \frac{90m}{6m}
\]

Because the demand has a negative slope \( b < 0 \) means that \( b = -4 \) and the supply has a has a positive slope \( h > 0 \) means that \( h = 2 \). Therefore, we can achieve that the changeable stability conditions is obtained. (Edward, 1994)

Discussion

The first-order linear differential equation can be used in different parts of the economics, so that in the first step, using the guided differential equation, then we can solve it by using the first-order linear differential equation solution.

In solving these issues, we can use replacements for ease of work and in other ways, the first change can be reached.

The first-order linear differential equation arises from the facts that we face in the social processes with such concepts and theories that you are required to establish relationships between dependent and free or non-dependent (independent) changes in the form of mathematical and snobisms, In some states, there is no possibility of the relationship between functions and variables directly and it is even unlikely, but it can be easily defined between its functions and its variables and the derivatives of the desired functions from different order of the snobisms and the exact mathematical formulas, so that the formula in total concepts and rules of higher mathematics are referred to as differential equations.

Conclusion

As the result of this article I found that the relationship of differential equations with economics has been mostly closed and expanded, and solution of many issues in economics depends on formation and solving of differential equations, and also I realized that evaluating the first-order linear differential equations in economics is a mathematical phenomenon and the first-order linear differential equations are important in physically changing and principles of economics and in market balance. Needs for recognizing the principles, concepts, laws and use of differential equations come from those truths that in social practical processes we usually confront with those concepts and ideas that imply to study and formulate the relationship between the market balance and stable balances. I also found that in some states, formulating the direct relationship between market balance and stable balance is impossible and even it is unlikely, but easily we can define and formulate the relationship between market balance and stable balance by a mathematical formulating that this mathematical formulating is generally called differential equations.
REFERENCES


