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Nil Complementary Domination in Intutionistic Fuzzy Graph

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ABSTRACT
The author defines the complementary dominating set and its number in intutionistic fuzzy graph. The author discussed the order and enclave is obtained for some standard intutionistic fuzzy graph is derived.

Keyword: *Intutionistic fuzzy graph, degree, complementary nil dominating set, effective degree.*

1. INTRODUCTION
In 1999, Atanassov[1] introduced the concept of intutionistic fuzzy relation and intutionistic fuzzy graphs. Parvathi and Karunambigai [5] introduced the concept of intutionistic fuzzy graph and analyzed its components. Nagoor Gani and Sajitha Begum [11] define degree, order and size in intutionistic fuzzy graph and derive its some of their properties. Somasundaram [9] introduced the concept of domination in intutionistic fuzzy graph. Parvathi and Thamizhendhi [6] introduced the concept of domination number in intutionistic fuzzy graph. Tamizh Chelvam [10] introduced and analyzed the complementary nil dominating set in the crisp graph. The properties of nil complementary dominating graph in intutionistic fuzzy graph is discussed in this paper.

2. Preliminaries
Definition 2.1
A fuzzy graph $G=(\sigma,\mu)$ is a pair of functions $\sigma:V\to[0,1]$ and $\mu:V\times V\to[0,1]$, where for all $u,v\in V$ $\mu(u,v)=\sigma(u)\wedge\sigma(v)$.

Definition 2.2
Let $G=(V,E)$ be an intutionistic fuzzy graph, such that

(i) $V=\{v_1,v_2......v_n\}$ such that $\mu_1:V\to[0,1]$, $\nu_1:V\to[0,1]$ denote the degree of membership and non-membership of the element $v_i\in V$ respectively and $0\le\mu_1(v_i)+\nu_1(v_i)\le 1$ for every $v_i\in V$, (i=1,2,......n).

(ii) $E\subseteq V\times V$ where $\mu_2:V\times V\to[0,1]$ and $\nu_2:V\times V\to[0,1]$ are such that $\mu_2(v_i,v_j)\le\mu_1(v_i)\wedge\mu_1(v_j)$ and $\nu_2(v_i,v_j)\le\nu_1(v_i)\wedge\nu_1(v_j)$ respectively and $0\le\mu_2(v_i,v_j)+\nu_2(v_i,v_j)\le 1$

Definition 2.3
Let $G=(V,E)$, be an intutionistic fuzzy graph. The cardinality of G is defined to be

$$|G|=\left|\sum_{v_i\in V}\frac{1+\mu_1(v_i)-\nu_1(v_i)}{2}+\sum_{v_i\in V}\frac{1+\mu_2(e_{ij})-\nu_2(e_{ij})}{2}\right|$$

The vertex cardinality is defined as

$$|V|=\sum_{v_i\in V}\frac{1+\mu_1(v_i)-\nu_1(v_i)}{2}$$

The edge cardinality is defined as

$$|E|=\sum_{e_{ij}\in V}\frac{1+\mu_2(e_{ij})-\nu_2(e_{ij})}{2}$$

Definition 2.4
Let $G=(V,E)$ be an intutionistic fuzzy graph. The degree of a vertex v_i is defined by $d(v_i)=(d_\mu(v_i),d_\nu(v_i))$ where $d_\mu(v)=\sum_{i\ne j}\mu_{ij}$ and $d_\nu(v)=\sum_{i\ne j}\nu_{ij}$

Definition 2.5
The effective degree of a vertex v in intutionistic fuzzy graph, G = (V,E) is defined to be the sum of the

strong edge incident at v. It is denoted by $\delta_E(G)$ and $\Delta_E(G)$.

The minimum degree of G is $\delta_E(G) = \min(d_E(v)/v \in V)$.

The maximum degree of G is $\Delta_E(G) = \max(d_E(v)/v \in V)$

Two vertices v_i and v_j are said to be neighbourhood in intutionistic fuzzy graph if there is a strong edges between v_i and v_j.

Definition 2.6

An intutionistic fuzzy graph $H' = (V', E')$ is said to be an intutionistic fuzzy subgraph of $G = (V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. That is $\mu'_{1i} \leq \mu_{1i}$; $\nu'_{1i} \leq \nu_{1i}$ and $\mu'_{2ij} \leq \mu_{2ij}$; $\gamma'_{2ij} \leq \gamma_{2ij}$ for every i,j=1,2,.....n

Definition 2.7

Let $G = (V, E)$,be an intutionistic fuzzy graph. The complement of an intutionistic fuzzy graph is, denoted by $G = (\overline{V}, \overline{E})$, to be satisfied the following conditions.

(i) $\overline{V} = v$

(ii) $\overline{\mu_{1i}} = \mu_{1i}$ and $\overline{\nu_{1i}} = \nu_{1i}$ for all i=1,2,.....n

(iii) $\overline{\mu_{2ij}} = \min(\mu_{1i}, \mu_{1j}) - \mu_{2ij}$ and $\overline{\nu_{2ij}} = \min(\nu_{1i}, \nu_{1j}) - \nu_{2ij}$ for all i,j=1,2,...n

Definition 2.8

Let $G = (V, E)$ be an intutionistic fuzzy graph. A set $S \subset V$ is said to be a nil complementary dominating set of an intutionistic fuzzy graph of G, if S is a dominating set and its complement V-S is not a dominating set. The minimum scalar cardinality over all nil complementary dominating set is called a nil complementary domination number and it is denoted by γ_{ncd}, the corresponding minimum nil complementary dominating set is denote by γ_{ncd} -set.

Definition 2.9

Let $S \subset V$ in the connected intutionistic fuzzy graph $G = (V, E)$. A vertex $u \in S$ is said to be an enclave of S if $\mu_2(u,v) < \max[\mu_1(u), \mu_1(v)]$ and $\nu_2(u,v) < \min[\nu_1(u), \nu_1(v)]$ for all $v \in V - S$. (i.e) $N(u) \subseteq S$.

3. Nil complementary in IFG

Theorem 3.1

A dominating set S is a nil complementary dominating set if and only if it contains at least one enclave.

Proof

Let S be a nil complementary dominating set of a intutionistic fuzzy graph $G = (V, E)$. The V-S is not a dominating set which implies that there exit a vertex $u \in S$ such that $\mu_2(u,v) < \max[\mu_1(u), \mu_1(v)]$ and $\nu_2(u,v) < \min[\nu_1(u), \nu_1(v)]$ for all $v \in V - S$. Therefore u is an enclave of S. Hence S contain atleast one enclave.

Conversely, suppose the dominating set S contains enclaves. Without loss of generality let us take u be the enclave of S. (i.e) $\mu_2(u,v) < \max[\mu_1(u), \mu_1(v)]$ and $\nu_2(u,v) < \min[\nu_1(u), \nu_1(v)]$ for all $v \in V - S$. Hence V-S is not a dominating set. Hence dominating set S is nil complementary dominating set.

Theorem 3.2

If S is a nil complementary dominating set of an intutionistic fuzzy graph $G = (V, E)$, then there is a vertex $u \in S$ such that $S - \{u\}$ is a dominating set.

Proof

Let S be a nil complementary dominating set. since by theorem 3.1, every nil complementary dominating set if and only if it contains atleast one enclave. Let $u \in S$ be an enclave of s. Then $\mu_2(u,v) < \max[\mu_1(u), \mu_1(v)]$ and $\nu_2(u,v) < \min[\nu_1(u), \nu_1(v)]$ for all $v \in V - S$. Since G is connected intutionistic fuzzy graph, there exist atleast a vertex $w \in S$ such that $\mu_2(u,v) = \max[\mu_1(u), \mu_1(v)]$ and $\nu_2(u,v) = \min[\nu_1(u), \nu_1(v)]$ hence $S - \{u\}$ is a dominating set.

Theorem 3.3

A nil complementary dominating set in an intutionistic fuzzy graph $G = (V, E)$ is not a singleton.

Proof

Let S be a nil complementary dominating set. Since by theorem 3.1, every nil complementary dominating set if and only if it contains atleast one enclave. Let $u \in S$ be an enclave of s. Then $\mu_2(u,v) < \max[\mu_1(u), \mu_1(v)]$ and

$\nu_2(u,v) < \min[\nu_1(u), \nu_1(v)]$ for all $v \in V - S$. Suppose S contains only one vertex u, then it must be isolated in G. This is contradiction to connectedness. Hence nil complementary dominating set contains more than one vertex.

Conclusion

The nil complementary dominating set and its number in intutionistic fuzzy graph is defined. Some of the properties are derived in this paper. The future work will be carried on its application in social network.

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