



Contra μ - β -Generalized α -Continuous Mappings in Generalized Topological Spaces

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ABSTRACT

In this paper, we have introduced contra μ - β -generalized α -continuous maps and also introduced almost contra μ - β -generalized α -continuous maps in generalized topological spaces by using μ - β -generalized α -closed sets (briefly μ - β G α CS). Also we have introduced some of their basic properties.

Keywords: Generalized topology, generalized topological spaces, μ - α -closed sets, μ - β -generalized α -closed sets, μ - α -continuous, μ - β -generalized α -continuous, contra μ - α -continuous, almost contra μ - β -generalized α -continuous.

1. INTRODUCTION

In 1970, Levin [6] introduced the idea of continuous function. He also introduced the concepts of semi-open sets and semi-continuity [5] in a topological space. Mashhour [7] introduced and studied α -continuous function in topological spaces. The notation of μ - β -generalized α -closed sets (briefly μ - β G α CS) was defined and investigated by Kowsalya M and Jayanthi. D[4]. Jayanthi. D [2, 3] also introduced contra continuity and almost contra continuity on generalized topological spaces. In this paper, we have introduced contra μ - β -generalized α -continuous maps.

2. PRELIMINARIES

Let us recall the following definitions which are used in sequel.

Definition 2.1: [1] Let X be a nonempty set. A collection μ of subsets of X is a generalized topology (or briefly GT) on X if it satisfies the following:

1. $\emptyset, X \in \mu$ and
2. If $\{M_i : i \in I\} \subseteq \mu$, then $\cup_{i \in I} M_i \in \mu$.

If μ is a GT on X , then (X, μ) is called a generalized topological space (or briefly GTS) and the elements of μ are called μ -open sets and their complement are called μ -closed sets.

Definition 2.2: [1] Let (X, μ) be a GTS and let $A \subseteq X$. Then the μ -closure of A , denoted by $c_\mu(A)$, is the intersection of all μ -closed sets containing A .

Definition 2.3: [1] Let (X, μ) be a GTS and let $A \subseteq X$. Then the μ -interior of A , denoted by $i_\mu(A)$, is the union of all μ -open sets contained in A .

Definition 2.4: [1] Let (X, μ) be a GTS. A subset A of X is said to be

- i. μ -semi-closed set if $i_\mu(c_\mu(A)) \subseteq A$
- ii. μ -pre-closed set if $c_\mu(i_\mu(A)) \subseteq A$
- iii. μ - α -closed set if $c_\mu(i_\mu(c_\mu(A))) \subseteq A$
- iv. μ - β -closed set if $i_\mu(c_\mu(i_\mu(A))) \subseteq A$
- v. μ -regular-closed set if $A = c_\mu(i_\mu(A))$

Definition 2.5: [7] Let (X, μ_1) and (Y, μ_2) be GTSs. Then a mapping $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is called

- i. μ -Continuous mapping if $f^{-1}(A)$ is μ -closed in (X, μ_1) for each μ -closed in (Y, μ_2) .
- ii. μ -Semi-continuous mapping if $f^{-1}(A)$ is μ -semi-closed in (X, μ_1) for every μ -closed in (Y, μ_2) .
- iii. μ -pre-continuous mapping if $f^{-1}(A)$ is μ -pre-closed in (X, μ_1) for every μ -closed in (Y, μ_2) .
- iv. μ - α -continuous mapping if $f^{-1}(A)$ is μ - α -closed in (X, μ_1) for every μ -closed in (Y, μ_2) .
- v. μ - β -continuous mapping if $f^{-1}(A)$ is μ - β -closed in (X, μ_1) for every μ -closed in (Y, μ_2) .

Definition 2.6: [9] Let (X, μ_1) and (Y, μ_2) be GTSSs. Then a mapping $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is called

- contra μ -Continuous mapping if $f^{-1}(A)$ is μ -closed in (X, μ_1) for every μ -open in (Y, μ_2) .
- contra μ -semi continuous mappings if $f^{-1}(A)$ is μ -semi closed in (X, μ_1) for every μ -open in (Y, μ_2) .
- contra μ -pre-continuous mappings if $f^{-1}(A)$ is μ -pre closed in (X, μ_1) for every μ -regular closed set A of (Y, μ_2) .
- contra μ - α -continuous mapping if $f^{-1}(A)$ is μ - α -closed in (X, μ_1) for every μ -open in (Y, μ_2) .
- contra μ - β -continuous mapping if $f^{-1}(A)$ is μ - β -closed in (X, μ_1) for every μ -open in (Y, μ_2) .

Definition 2.7: [3] Let (X, μ_1) and (Y, μ_2) be GTSSs. Then a mapping $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is called

- almost contra μ -Continuous mapping if $f^{-1}(A)$ is μ -closed in (X, μ_1) for every μ -regular open in (Y, μ_2) .
- almost contra μ -semi continuous mappings if $f^{-1}(A)$ is μ -semi closed in (X, μ_1) for every μ -regular open in (Y, μ_2) .
- almost contra μ -pre-continuous mappings if $f^{-1}(A)$ is μ -pre closed in (X, μ_1) for every μ -regular open in (Y, μ_2) .
- almost contra μ - α -continuous mapping if $f^{-1}(A)$ is μ - α -closed in (X, μ_1) for every μ -regular open in (Y, μ_2) .
- Almost contra μ - β -continuous mapping if $f^{-1}(A)$ is μ - β -closed in (X, μ_1) for every μ -regular open in (Y, μ_2) .

3. CONTRA μ - β -GENERALIZED α -CONTINUOUS MAPPINGS

In this chapter we have introduced contra μ - β -generalized α -continuous mapping in generalized topological spaces and studied their properties.

Definition 3.1: A mapping $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is called a contra μ - β -generalized α -continuous mapping if $f^{-1}(A)$ is a μ - β -generalized α -closed set in (X, μ_1) for each μ -open set A in (Y, μ_2) .

Example 3.2: Let $X = Y = \{a, b, c\}$ with $\mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{c\}, Y\}$. Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by $f(a) = a, f(b) = b, f(c) = c$. Now,

$$\mu\text{-}\beta\text{O}(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}.$$

Let $A = \{c\}$, then A is a μ -open set in (Y, μ_2) . Then $f^{-1}(\{c\})$ is a μ - β -generalized α -closed set in (X, μ_1) . Hence f is a contra μ - β -generalized α -continuous mapping.

Theorem 3.3: Every contra μ -continuous mapping is a contra μ - β -generalized α -continuous mapping but not conversely in general.

Proof: Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a contra μ -continuous mapping. Let A be any μ -open set in (Y, μ_2) . Since f is a contra μ -continuous mapping, $f^{-1}(A)$ is a μ -closed set in (X, μ_1) . Since every μ -closed set is a μ - β -generalized α -closed set, $f^{-1}(A)$ is a μ - β -generalized α -closed set in (X, μ_1) . Hence f is a contra μ - β -generalized α -continuous mapping.

Example 5.1.4: Let $X = Y = \{a, b, c, d\}$ with $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ and $\mu_2 = \{\emptyset, \{d\}, Y\}$. Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by $f(a) = a, f(b) = b, f(c) = c, f(d) = d$. Now,

$$\mu\text{-}\beta\text{O}(X) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, X\}.$$

Let $A = \{d\}$, then A is a μ -open set in (Y, μ_2) . Then $f^{-1}(\{d\})$ is a μ - β -generalized α -closed set, but not μ -closed as $c_\mu(f^{-1}(A)) = c_\mu(\{d\}) = \{b, d\} \neq f^{-1}(A)$ in (X, μ_1) . Hence f is a contra μ - β -generalized α -continuous mapping, but not a contra μ -continuous mapping.

Theorem 3.5: Every contra μ - α -continuous mapping is a contra μ - β -generalized α -continuous mapping in general.

Proof: Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a μ - α -contra continuous mapping. Let A be any μ -open set in (Y, μ_2) . Since f is a μ - α -contra continuous mapping, $f^{-1}(A)$ is a μ - α -closed set in (X, μ_1) . Since every μ - α -closed set is a μ - β -generalized α -closed set, $f^{-1}(A)$ is a μ - β -generalized α -closed set in (X, μ_1) . Hence f is a contra μ - β -generalized α -continuous mapping.

Remark 3.6: A contra μ -pre-continuous mapping is not a contra μ - β -generalized α -continuous mapping in general.

Example 3.7: Let $X = Y = \{a, b, c\}$ with $\mu_1 = \{\emptyset, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{a\}, Y\}$. Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by $f(a) = a, f(b) = b, f(c) = c$. Now,

$$\mu\text{-}\beta\text{O}(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}.$$

Let $A = \{a\}$, then A is a μ -open set in (Y, μ_2) . Then $f^{-1}(\{a\})$ is a μ -pre closed set as $c_\mu(i_\mu(f^{-1}(A))) = c_\mu(i_\mu(\{a\})) = \emptyset \subseteq f^{-1}(A)$, but not a μ - β -generalized α -closed set as $\alpha c_\mu(f^{-1}(A)) = X \not\subseteq U = \{a, b\}$ in (X, μ_1) . Hence f is a contra μ -pre-continuous mapping, but not a contra μ - β -generalized α -continuous mapping.

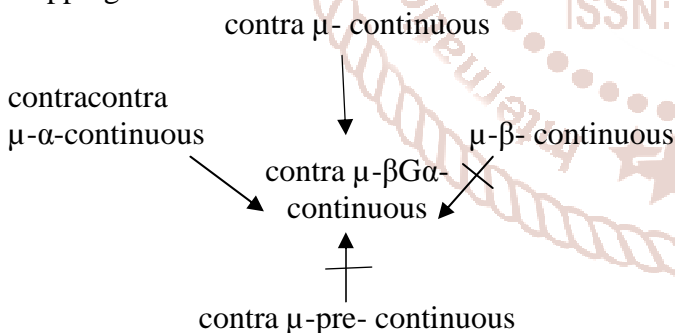
Remark 3.8: A contra μ - β -continuous mapping is not a contra μ - β -generalized α -continuous mapping in general.

Example 3.9: Let $X = Y = \{a, b, c\}$ with $\mu_1 = \{\emptyset, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{a\}, Y\}$. Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by $f(a) = a, f(b) = b, f(c) = c$. Now,

$$\mu\text{-}\beta\text{O}(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}.$$

Let $A = \{a\}$, then A is a μ -open set in (Y, μ_2) . Then $f^{-1}(\{a\})$ is a μ - β -closed set as $i_\mu(c_\mu(i_\mu(f^{-1}(A)))) = i_\mu(c_\mu(i_\mu(\{a\}))) = \emptyset \subseteq f^{-1}(A)$, but not μ - β -generalized α -closed set $\alpha c_\mu(f^{-1}(A)) = X \not\subseteq U = \{a, b\}$ in (X, μ_1) . Hence f is a contra μ - β -continuous mapping, but not a contra μ - β -generalized α -continuous mapping.

In the following diagram, we have provided the relation between various types of contra μ -continuous mappings.



Theorem 3.10: A mapping $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is a contra μ - β -generalized α -continuous mapping if and only if the inverse image of every μ -closed set in (Y, μ_2) is a μ - β -generalized α -open set in (X, μ_1) .

Proof: Necessity: Let F be a μ -closed set in (Y, μ_2) . Then $Y-F$ is a μ -open in (Y, μ_2) . Then $f^{-1}(Y-F)$ is a μ - β -generalized α -closed set in (X, μ_1) , by hypothesis. Since $f^{-1}(Y-F) = X - f^{-1}(F)$. Hence $f^{-1}(F)$ is a μ - β -generalized α -open set in (X, μ_1) .

Sufficiency: Let F be a μ -open set in (Y, μ_2) . Then $Y-F$ is a μ -closed in (Y, μ_2) . By hypothesis, $f^{-1}(Y-F)$ is a μ - β -generalized α -open set in (X, μ_1) . Since $f^{-1}(Y-F) = X - f^{-1}(F)$ is a μ - β -generalized α -open set in (X, μ_1) . Therefore $f^{-1}(F)$ is a μ - β -generalized α -closed set in (X, μ_1) . Hence f is a contra μ - β -generalized α -continuous mapping.

Theorem 3.11: Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping and let $f^{-1}(V)$ be a μ -open set in (X, μ_2) for every closed V set in (Y, μ_2) . Then f is a contra μ - β -generalized α -continuous mapping.

Proof: Let V be a μ -closed set in (Y, μ_2) . Then $f^{-1}(V)$ be a μ -open set in (X, μ_1) , by hypothesis. Since every μ -open set is μ - β -generalized α -open set in X . Hence $f^{-1}(V)$ is a μ - β -generalized α -open set in (X, μ_1) . Hence f is a contra μ - β -generalized α -continuous mapping.

Theorem 3.12: If $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is a contra μ - β -generalized α -continuous mapping and $g: (Y, \mu_2) \rightarrow (Z, \mu_3)$ is a μ -continuous mapping then $g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3)$ is a contra μ - β -generalized α -continuous mapping.

Proof: Let V be any μ -open set in (Z, μ_3) . Then $g^{-1}(V)$ is a μ -open set in (Y, μ_2) , since g is a μ -continuous mapping. Since f is a contra μ - β -generalized α -continuous mapping, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a μ - β -generalized α -closed set in (X, μ_1) . Therefore $g \circ f$ is a contra μ - β -generalized α -continuous mapping.

Theorem 3.13: If $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is a contra μ -continuous mapping and $g: (Y, \mu_2) \rightarrow (Z, \mu_3)$ is a contra μ -continuous mapping then $g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3)$ is a μ - β -generalized α -continuous mapping.

Proof: Let V be any μ -open set in (Z, μ_3) . Since g is a contra μ -continuous mapping, $g^{-1}(V)$ is a μ -closed set in (Y, μ_2) . Since f is a contra μ -continuous mapping, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a μ -open set in (X, μ_1) . Since every μ -open set is a μ - β -generalized α -open set, $(g \circ f)^{-1}(V)$ is a μ - β -generalized α -open set in (X, μ_1) . Therefore $g \circ f$ is a μ - β -generalized α -continuous mapping.

Theorem 3.14: If $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is a contra μ - α -continuous mapping and $g: (Y, \mu_2) \rightarrow (Z, \mu_3)$ is a contra μ -continuous mapping then $g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3)$ is a μ - β -generalized α -continuous mapping.

Proof: Let V be any μ -closed set in (Z, μ_3) . Since g is a μ -contra continuous mapping, $g^{-1}(V)$ is a μ -open set in (Y, μ_2) . Since f is a μ - α -contra continuous mapping, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a μ - α -closed set in (X, μ_1) . Since every μ - α -closed set is a μ - β -generalized α -closed set, $(g \circ f)^{-1}(V)$ is a μ - β -generalized α -closed set in (X, μ_1) . Therefore $g \circ f$ is a μ - β -generalized α -continuous mapping.

Theorem 3.15: If $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is a μ -continuous mapping and $g: (Y, \mu_2) \rightarrow (Z, \mu_3)$ is a contra μ -continuous mapping then $g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3)$ is a contra μ - β -generalized α -continuous mapping.

Proof: Let V be any μ -open set in (Z, μ_3) . Since g is a contra μ -continuous mapping, $g^{-1}(V)$ is a μ -closed set in (Y, μ_2) . Since f is μ -continuous $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a μ -closed set in (X, μ_1) . Since every μ -closed set is a μ - β -generalized α -closed set, $(g \circ f)^{-1}(V)$ is a μ - β -generalized α -closed set. Therefore $g \circ f$ is a contra μ - β -generalized α -continuous mapping.

4. ALMOST CONTRA μ - β -GENERALIZED α -CONTINUOUS MAPPINGS

In this section we have introduced almost contra μ - β -generalized α -continuous mapping in generalized topological spaces and studied some of their basic properties.

Definition 4.1: A mapping $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is called an almost contra μ - β -generalized α -continuous mapping if $f^{-1}(A)$ is a μ - β -generalized α -closed set in (X, μ_1) for each μ -regular open set A in (Y, μ_2) .

Example 4.2: Let $X = Y = \{a, b, c\}$ with $\mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{c\}, \{a, b\}, Y\}$. Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by $f(a) = a, f(b) = b, f(c) = c$. Now,
 μ - β O(X) = $\{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$.

Let $A = \{c\}$, then A is a μ -regular open set in (Y, μ_2) . Then $f^{-1}(\{c\})$ is a μ - β -generalized α -closed set in (X, μ_1) . Hence f is an almost contra μ - β -generalized α -continuous mapping.

Theorem 4.3: Every almost contra μ -continuous mapping is an almost contra μ - β -generalized α -continuous mapping but not conversely.

Proof: Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be an almost contra μ -continuous mapping. Let A be any μ -regular open set

in (Y, μ_2) . Since f is almost contra μ -continuous mapping, $f^{-1}(A)$ is a μ -closed set in (X, μ_1) . Since every μ -closed set is a μ - β -generalized α -closed set, $f^{-1}(A)$ is a μ - β -generalized α -closed set in (X, μ_1) . Hence f is an almost contra μ - β -generalized α -continuous mapping.

Example 4.4: Let $X = Y = \{a, b, c, d\}$ with $\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ and $\mu_2 = \{\emptyset, \{d\}, \{a, b, c\}, Y\}$. Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by $f(a) = a, f(b) = b, f(c) = c, f(d) = d$. Now,

μ - β O(X) = $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, X\}$.

Let $A = \{d\}$, then A is a μ -regular open set in (Y, μ_2) . Then $f^{-1}(\{d\})$ is a μ - β -generalized α -closed set, but not μ -closed as $c_\mu(f^{-1}(A)) = c_\mu(\{d\}) = \{b, d\} \neq f^{-1}(A)$ in (X, μ_1) . Hence f is an almost contra μ - β -generalized α -continuous mapping, but not almost contra μ -continuous mapping.

Theorem 4.5: Every almost contra μ - α -continuous mapping is an almost contra μ - β -generalized α -continuous mapping in general.

Proof: Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be an almost contra μ - α -continuous mapping. Let A be any μ -regular open set in (Y, μ_2) . Since f is an almost contra μ - α -continuous mapping, $f^{-1}(A)$ is a μ - α -closed set in (X, μ_1) . Since every μ - α -closed set is a μ - β -generalized α -closed set, $f^{-1}(A)$ is μ - β -generalized α -closed set in (X, μ_1) . Hence f is an almost contra μ - β -generalized α -continuous mapping.

Remark 4.6: An almost contra μ -pre-continuous mapping is not an almost contra μ - β -generalized α -continuous mapping in general.

Example 4.7: Let $X = Y = \{a, b, c\}$ with $\mu_1 = \{\emptyset, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by $f(a) = a, f(b) = b, f(c) = c$. Now,

μ - β O(X) = $\{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$.

Let $A = \{a\}$, then A is a μ -regular open set in (Y, μ_2) . Then $f^{-1}(\{a\})$ is a μ -pre closed as $c_\mu(i_\mu(f^{-1}(A))) = c_\mu(i_\mu(\{a\})) = \emptyset \subseteq f^{-1}(A)$, but not μ - β -generalized α -closed set as $\alpha c_\mu(f^{-1}(A)) = X \not\subseteq U = \{a, b\}$ in (X, μ_1) . Hence f is an almost contra μ -pre-continuous

mapping, but not an almost contra μ - β -generalized α -continuous mapping.

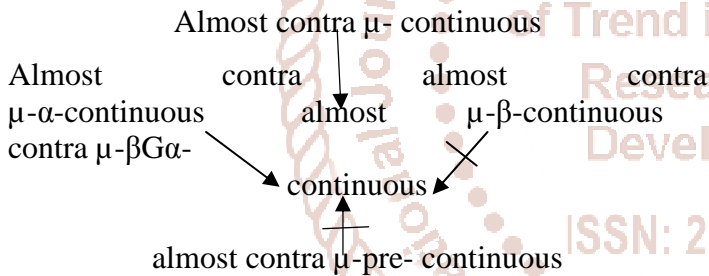
Remark 4.8: An almost contra μ - β -continuous mapping is not an almost contra μ - β -generalized α -continuous mapping in general.

Example 4.9: Let $X = Y = \{a, b, c\}$ with $\mu_1 = \{\emptyset, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Let $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ be a mapping defined by $f(a) = a, f(b) = b, f(c) = c$. Now,

$$\mu\text{-}\beta\text{O}(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}.$$

Let $A = \{a\}$, then A is a μ -regular open set in (Y, μ_2) . Then $f^{-1}(\{a\})$ is a μ - β -closed set as $i_\mu(c_\mu(i_\mu(f^{-1}(A)))) = i_\mu(c_\mu(i_\mu(\{a\}))) = \emptyset \subseteq f^{-1}(A)$, but not a μ - β -generalized α -closed set as $\alpha c_\mu(f^{-1}(A)) = X \not\subseteq U = \{a, b\}$ in (X, μ_1) . Hence f is an almost contra μ - β -continuous mapping, but not almost contra μ - β -generalized α -continuous mapping.

In the following diagram, we have provided the relation between various types of almost contra μ -continuous mappings.



Theorem 4.10: A mapping $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is an almost contra μ - β -generalized α -continuous mapping if and only if the inverse image of every μ -regular closed set in (Y, μ_2) is a μ - β -generalized α -open set in (X, μ_1) .

Proof:

Necessity: Let F be a μ -regular closed set in (Y, μ_2) . Then $Y-F$ is a μ -regular open in (Y, μ_2) . Since f is an almost contra μ - β -generalized α -continuous, $f^{-1}(Y-F)$ is μ - β -generalized α -closed set in (X, μ_1) . Since $f^{-1}(Y-F) = X - f^{-1}(F)$. Hence $f^{-1}(F)$ is μ - β -generalized α -open set in (X, μ_1) .

Sufficiency: Let F be a μ -regular open set in (Y, μ_2) . Then $Y-F$ is a μ -regular closed in (Y, μ_2) . By hypothesis, $f^{-1}(Y-F)$ is a μ - β -generalized α -open set in (X, μ_1) . Since $f^{-1}(Y-F) = X - f^{-1}(F)$ is a μ - β -generalized α -open set, $f^{-1}(F)$ is a μ - β -generalized α -

closed set in (X, μ_1) . Hence f is an almost contra μ - β -generalized α -continuous mapping.

Theorem 4.11: If $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is a μ -continuous mapping and $g: (Y, \mu_2) \rightarrow (Z, \mu_3)$ is an almost contra μ -continuous mapping then $g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3)$ is an almost contra μ - β -generalized α -continuous mapping.

Proof: Let V be any μ -regular open set in (Z, μ_3) . Since g is an almost contra μ -continuous mapping, $g^{-1}(V)$ is a μ -closed set in (Y, μ_2) . Since f is a μ -continuous mapping, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a μ -closed in (X, μ_1) . Since every μ -closed set is a μ - β -generalized α -closed set, $(g \circ f)^{-1}(V)$ is a μ - β -generalized α -closed set in (X, μ_1) . Therefore $g \circ f$ is an almost contra μ - β -generalized α -continuous mapping.

Theorem 4.12: If $f: (X, \mu_1) \rightarrow (Y, \mu_2)$ is a μ - α -continuous mapping and $g: (Y, \mu_2) \rightarrow (Z, \mu_3)$ is an almost contra μ -continuous mapping then $g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3)$ is a contra μ - β -generalized α -continuous mapping.

REFERENCE

1. **Csaszar, A.**, Generalized topology, generalized continuity, Acta Mathematica Hungar., 96 (4) (2002), 351 - 357.
2. **Jayanthi, D.**, Contra continuity on generalized topological spaces, Acta. Math. Hungar., 134(2012), 263-271.
3. **Jayanthi, D.**, almost Contra continuity on generalized topological spaces, Indian, journal of research., 12(2013), 15-21.
4. **Kowsalya, M. And Jayanthi, D.**, μ - β generalized α -closed sets in generalized topological spaces (submitted).
5. **Levine, N.**, Semi open sets and semi continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36-41.
6. **Levine, N.**, Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19 (1970), 89-96.
7. **Mashhour, A. S., Hasanein, I. A., and El-Deeb S. N.**, α -continuous and α -open mappings, Acta. Math. Hungar, 41 (1983), no. 3-4, 213-218.
8. **Min. W. K.**, Almost continuity on generalized topological spaces, Acta. Math. Hungar., 125 (1-2) (2009), 121-125.
9. **Min. W. K.**, Generalized continuous functions defined by generalized open sets on generalized topological spaces, Acta. Math. Hungar., 2009.