



A Study on Minimal Spanning Tree on Network Flow

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ABSTRACT

In this paper we discussed the concept the characteristic and application of minimal spanning tree and how it is applied on network areas, and also we solved some real life problems.

KEYWORD: Network flow, Nodes, Links, Minimal spanning tree

INTRODUCTION

Trees are nothing more than restricted types of graphs, just with many more rules to follow. A tree will always be a graph, but not all graphs will be trees. A vertex is also referred to as a node, a junction, a point 0 – cell or an 0 simplex. Instead of shortest spanning tree, one may wish to find a longest spanning tree or one may be interested the trees with other desired properties and constraints. Such as, a spanning tree with specified maximum degree or diameter.

Characteristics:

- We are given the nodes of a network but not the link instead we are given the potential links between node pairs and the length for each potential link.
- It is desired to choose enough potential links in such a way that there is a part connecting all the nodes of the network.
- That is no node is left out unconnected.
- The choosen links must provide a path between each pair of node in such a way that the total length of these links is a minimum.

Application of minimal spanning Tree:

- **Telecommunication networks:** In these age of information super highway application in this area becomes, more important as many telecommunication network are highly expensive

with is very important to optimize their designs by finding minimum spanning trees for them.

- **Power Transmission Network:** To provide high voltage power transmission lines.
- **Pipeline Network:** To connect sources to a no of destination.
- **Transportation Network:** To provide links at minimum cost.

Algorithm:

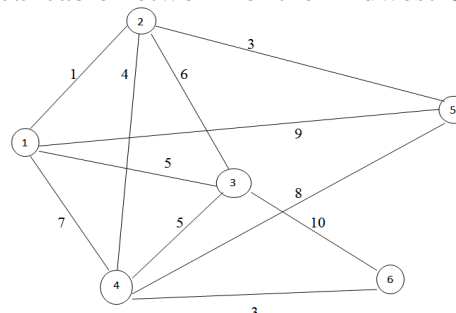
Step 1: start with any given node. Connect it to the nearest distinct node. Tick all the nodes that have been connected.

Step 2: Identify the un connect node that is closest to a connected node and then connect these two nodes.

Step 3: Repeat step2 untill all nodes are connected. The resulting network is guaranteed to be a minimal spanning tree they may be ties for the nearest distinct node or the closest unconnected node. In such a cases break the ties arbitrarily the algorithm will still yield an optimal solution.

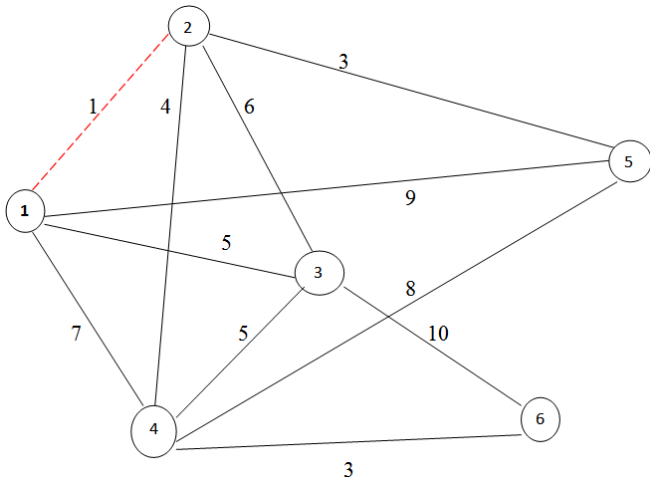
General Problem:

The Midwest TV cable company is in the process of providing cable services to five new housing development areas. The figure below depict potential tv linkages among the five areas. The cable miles are shown on each branch. Determine the most economical cable network for the Midwest Company.

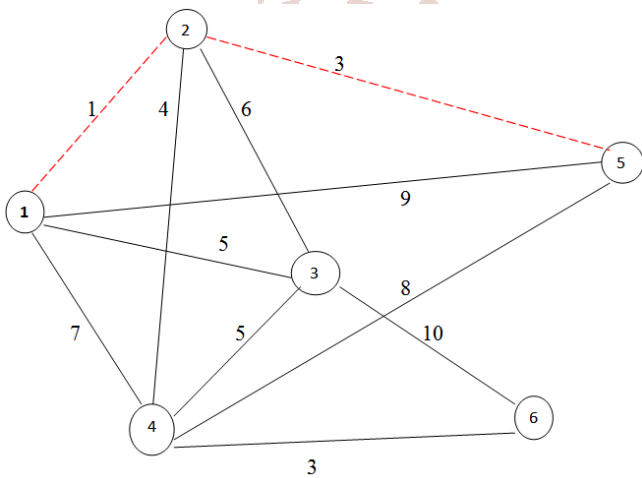


SOLUTION

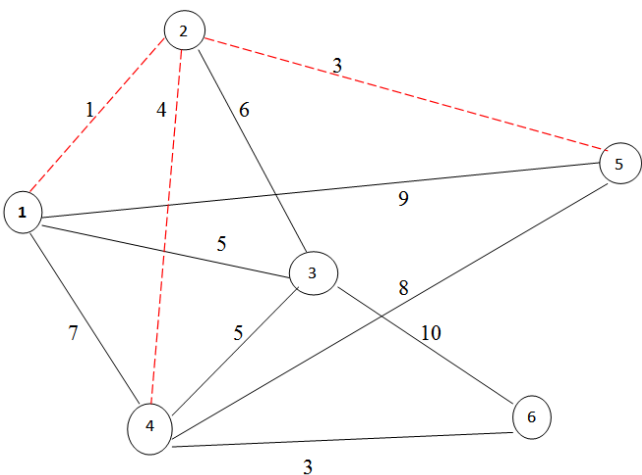
Step 1: We start with node 1. Connected to the nearest distinct node 2. color node 1 and 2 as both are now connected



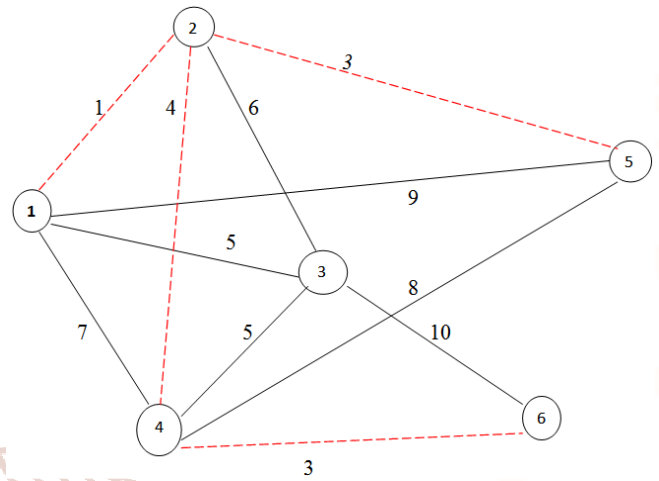
Step 2: identify the unconnected node that is closest to a connected node. The unconnected node closest to either nodes 1 or 2 is node 5. So, we connect node 2 to node 5 and color it to indicate that is connected node.



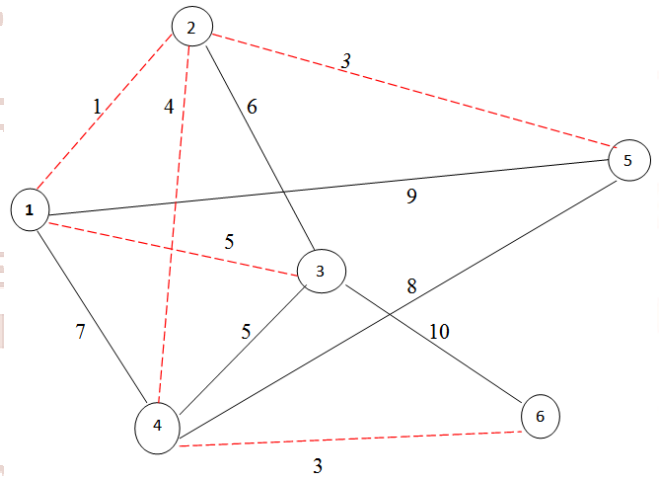
Step 3: the unconnected node closest any colored node is node 4. So we connected to node 4 to node 2.



Step 4: The unconnected node closest to colored node is node 6. So we connect node 6 to node 4.



Step 5: The unconnected node closest to colored node is node 3. So we connect node 3 to node 1.



Step 6: All nodes are now connected. Thus, the minimal spanning tree provides an optimal solution.

The most economical cable network i.e. the minimal spanning tree for the Midwest Company given by the links connecting the node pairs as indicated.

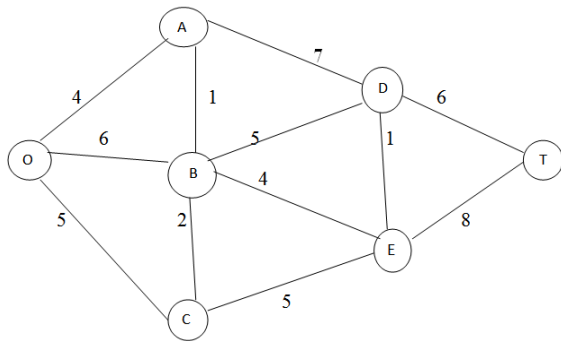
$$1-2-5; 2-4-6; 1-3.$$

The minimum cable miles needed for this spanning tree is given by

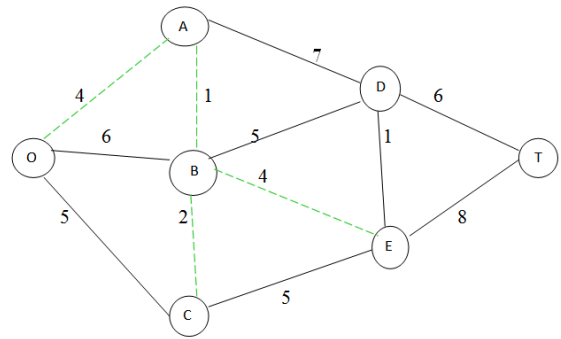
$$Z = 1+3+4+3+5 = 16 \text{ miles.}$$

PROBLEMS

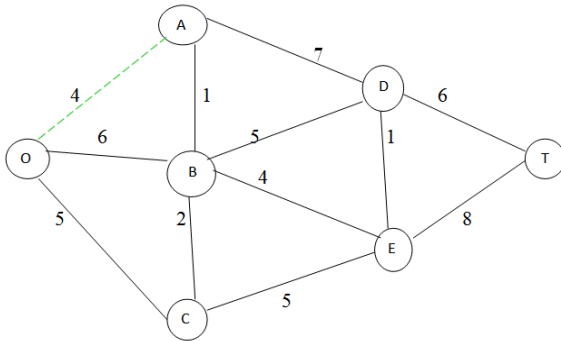
Consider the following network where the number on links represent actual distance between the corresponding nodes. Find the minimal spanning tree



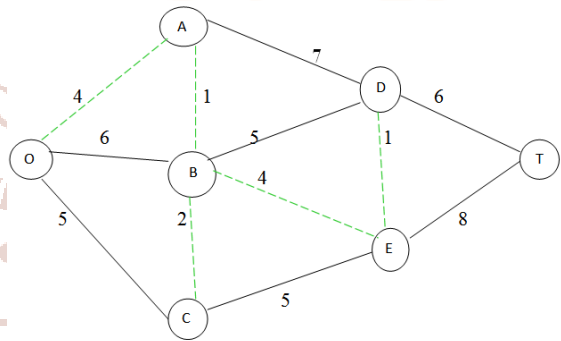
Step 1: We start with node O. connect it to the nearest distinct node A. Colour the nodes O and A as both are now connected nodes.



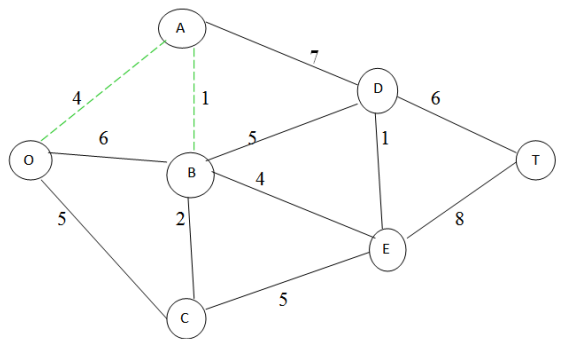
Step 5: The unconnected node closest to a coloured node is node D. So, we connect node D to node E.



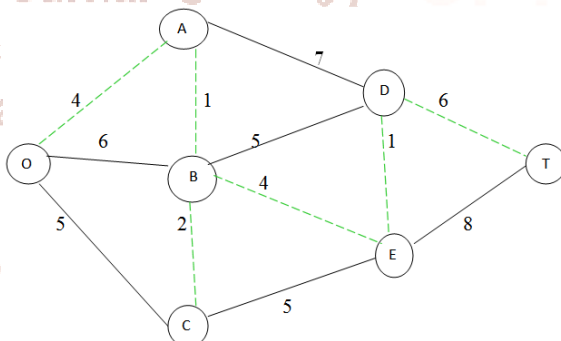
Step 2: Identify the unconnected node that is closest to a connected node. The unconnected node closest to either node O or node A is node B. So, we connect node A to node B and colour it to indicate that is connected node.



Step 6: The unconnected node closest to coloured node is node T. So, we connect node T to node D.



Step 3: The unconnected node closest to any coloured node is node C. So, we connect node C to node B.



Step 7: All nodes are connected. Thus the minimal spanning tree provide an optimal solution.

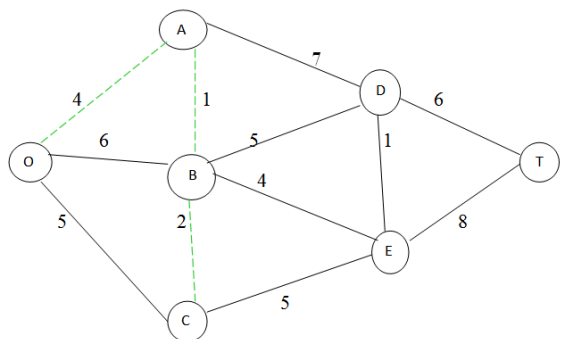
The most economical cable network, i.e. the minimal spanning tree for network is given by the link connecting the node pairs as indicated :

O-A-B-C; B-E-D-T

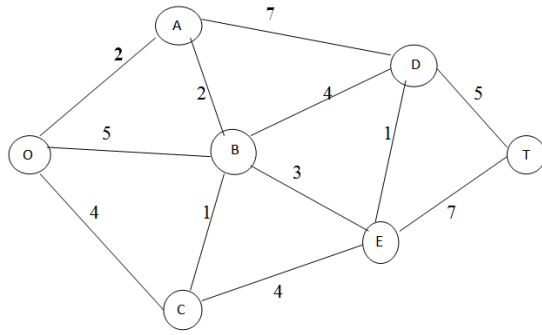
The minimal cable miles needed for this spanning tree is

$$Z = 4+1+2+4+1+6 = 18 \text{ miles}$$

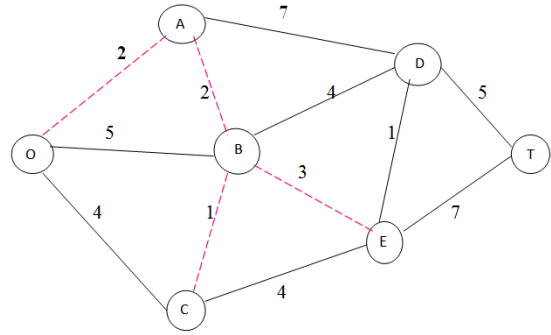
The management of city information park desires to determine which road telephone cable should be install to connect all 7 work station with a minimal total length of the cable .Node and distance of potential links are given in the network below . Use the minimal spanning tree algorithm to find the most economical cable tree:



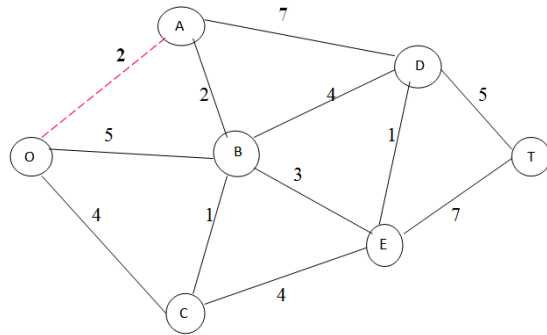
Step 4: The unconnected node closest to any coloured node is node E. So, we connect node E to node B.



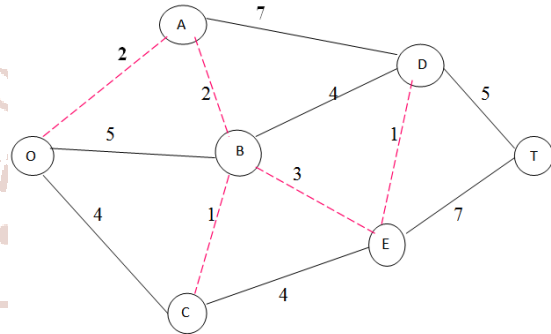
Step 1: We start with node O. connect it to the nearest distinct node A .Colour the nodes O and A as both are now connected nodes.



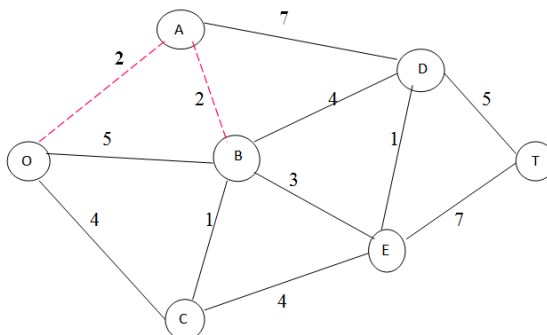
Step 5: The unconnected node closest to a coloured node is node D. So, we connect node D to node E.



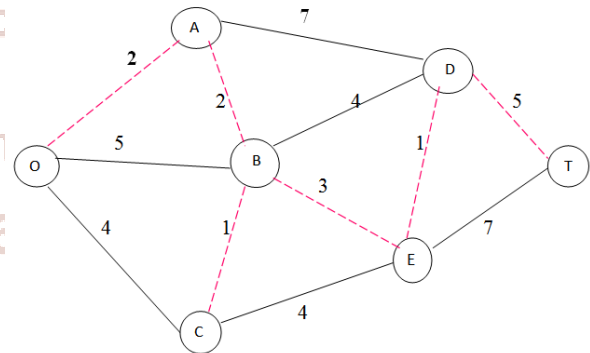
Step 2: Identify the unconnected node that is closest to a connected node. The unconnected node closest to either node O or node A is node B. So, we connect node A to node B and colour it to indicate that is connected node.



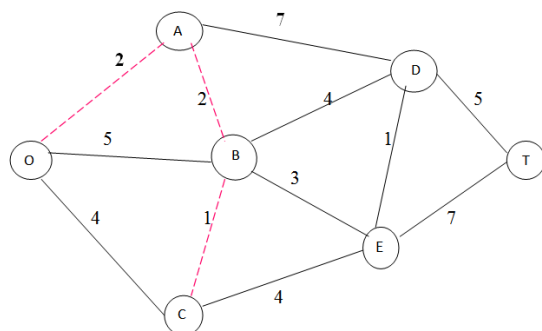
Step 6: The unconnected node closest to coloured node is node T. So, we connect node T to node D.



Step 3: The unconnected node closest to any coloured node is node C. So, we connect node C to node B.



Step 7: All nodes are connected .Thus the minimal spanning tree provide an optimal solution.



Step 4: The unconnected node closest to any coloured node is node E. So, we connect node E to node B.

The most economical cable network, i.e. the minimal spanning tree for network is given by the link connecting the node pairs as indicated:

O-A-B-C; B-E-D-T

The minimal cable miles needed for this spanning tree is

$$Z = 2+2+1+3+1+5 = 14 \text{ miles}$$

Conclusions:

These problems can be expressed and solved elegantly as graph theory problems involving connected and weighted digraph. From a practical point of view, all these problems are trivial if the network is small. Many real life situation however

consist of huge networks, and therefore it is important to look at these network problems in terms of solving them on computers.

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