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Reliability Modeling and Analysis of a Parallel Unit System with Priority to Repair over Replacement Subject to Maximum Operation and Repair Times

Reetu Rathee[1], D. Pawar[1], S. C. Malik[2]
[1]Assistant Professor, [2]Professor
[1]Amity Institute of Applied Science, AMITY University, Noida, Uttar Pradesh, India
[2]Department of Statistics, M. D. University, Rohtak, Haryana, India

ABSTRACT
This paper proposes the study of modelling and analysis of a two unit parallel system. A constant failure rate is considered for the units which are identical in nature. All repair activities like repair, replacement, preventive maintenance are mended immediately by a single server. The repair of the unit is done after its failure and if the fault is not rectified by the server within a given repair time, called maximum repair time, the unit replaced by new one. And, if there is no fault occurs up to a pre-fixed operation time, called maximum operation time, the unit undergoes for the preventive maintenance. The unit works as new after all repair activities done by the server. Priority to repair of one unit is given over the replacement of the other one. All random variables are statistically independent. The distribution for the failure, preventive maintenance and replacement rates are negative exponential whereas the distribution for all repair activities are taken as arbitrary with different probability density functions. Semi-Markov and regenerative point techniques are used to derive some reliability measures in steady state. The variation of MTSF, availability and profit function has been observed graphically for various parameters and costs.

Keywords: *Parallel system, Preventive Maintenance, Replacement, Priority, Reliability Measures*

INTRODUCTION
The general purpose of the modern world is to achieve the require performance level using the lowest possible cost. And, the parallel system works not only for maximize the profit but also for minimize the failure risk as well as cost. Keeping in view of their practical applications, reliability models of parallel systems have been developed and analyzed stochastically by the researchers and reliability engineers. Kishan and Kumar (2009) evaluated stochastically a parallel system using preventive maintenance. Further, kumar et al. (2010) and Malik and Gitanjali (2012) have analyzed cost-benefit of a parallel system subject to degradation after repair and arrival time of the server respectively. However, to enhance the profit of the system Reetu and Malik (2013) and Rathee and Chander (2014) developed parallel systems using the concept of priority.

Also, the objective of the present paper is to determine the reliability measures by giving the priority to one repair activity over the other ones. A constant failure rate is considered for the units which are identical in nature. All repair activities are done immediately by a single server. The repair of the unit is conducted after its failure and if the fault is not rectified by the server within a given repair time, the unit replaced by new one. And, if there is no fault occurs up to a pre-fixed operation time, the preventive maintenance is conducted. The unit works as new after all repair activities done by the server. Priority to repair of one unit is given over the replacement of the other one. All random variables are statistically independent. The distribution for the failure, preventive maintenance and replacement rates are negative exponential whereas the distribution for all repair activities are taken as arbitrary with different probability density functions. Semi-Markov and regenerative point techniques are used to derive some reliability measures in steady state. The variation of MTSF, availability and profit function has been observed graphically for various parameters and costs.

NOTATIONS:

E : Set of regenerative states

$\bar{E}$: Set of non-regenerative states

λ : Constant failure rate

α_0 : The rate by which system undergoes for preventive maintenance (called maximum constant rate of operation time)

β_0 : The rate by which system undergoes for replacement (called maximum constant rate of repair time)

FUr /FWr : The unit is failed and under repair/waiting for repair

FURp/FWRp : The unit is failed and under replacement/waiting for replacement

UPm/WPm : The unit is under preventive maintenance/waiting for preventive maintenance

FUR/FWR : The unit is failed and under repair / waiting for repair continuously from previous state

FURP/FWRP : The unit is failed and under /waiting for replacement continuously from previous state

UPM/WPM : The unit is under/waiting for preventive maintenance continuously from previous state

g(t)/G(t) : pdf/cdf of repair time of the unit

f(t)/F(t) : pdf/cdf of preventive maintenance time of the unit

r(t)/R(t) : pdf/cdf of replacement time of the unit

$q_{ij}(t)/Q_{ij}(t)$: pdf / cdf of passage time from regenerative state Si to a regenerative stateSjor to a failed state Sj without visiting any other regenerative state in (0, t]

$q_{ij.kr}(t)/Q_{ij.kr}(t)$: pdf/cdf of direct transition time from regenerative state Si to a regenerative state Sj or to a failed state Sj visiting state Sk, Sr once in (0, t]

$M_i(t)$: Probability that the system up initially in state $S_i \in$ E is up at time t without visiting to any regenerative state

$W_i(t)$: Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.

μ_i : The mean sojourn time in state S_i which is given by

$$\mu_i = E(T) = \int_0^\infty P(T > t)\, dt = \sum_j m_{ij},$$

where T denotes the time to system failure.

m_{ij}:Contribution to mean sojourn time (μ_i) in state S_i when system transits directlyto state $S_j$so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$

&/© : Symbol for Laplace-Stieltjes convolution/Laplace convolution

*/** : Symbol for Laplace Transformation /LaplaceStieltjes Transformation

The states S_0, S_1, S_2, S_4, $S_6$and S_7 are regenerative while the states S_3,S_5, S_8,S_9, S_{10}, S_{11} and S_{12} are non-regenerative as shown in figure 1.

TRANSITION STATE DIAGRAM

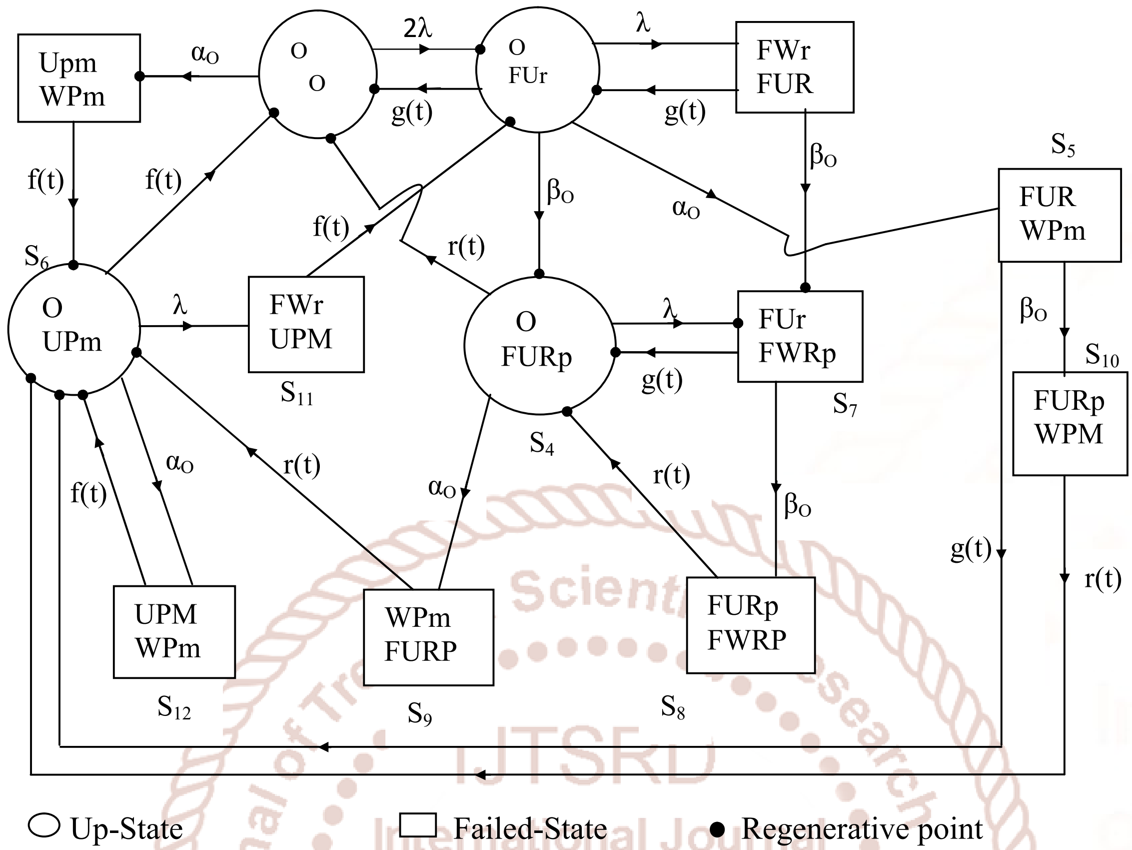

Fig. 1

Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$ as (1)

$$p_{01} = \frac{2\lambda}{2\lambda+\alpha_0},\ p_{02} = \frac{\alpha_0}{2\lambda+\alpha_0},\ p_{15} = \frac{\alpha_0}{(\lambda+\alpha_0+\beta_0)}(1-g^*(\lambda+\alpha_0+\beta_0)),$$

$$p_{40} = r^*(\lambda+\alpha_0),\ p_{13} = \frac{\lambda}{(\lambda+\alpha_0+\beta_0)}(1-g^*(\lambda+\alpha_0+\beta_0)),\ p_{60} = f^*(\lambda+\alpha_0),$$

$$p_{10} = g^*(\lambda+\alpha_0+\beta_0),\ p_{17.3} = \frac{\lambda}{(\lambda+\alpha_0+\beta_0)}(1-g^*(\beta_0))(1-g^*(\lambda+\alpha_0+\beta_0)),$$

$$p_{6,12} = p_{66.12} = \frac{\alpha_0}{(\lambda+\alpha_0)}(1-f^*(\lambda+\alpha_0)),\ p_{6,11} = p_{61.11} = \frac{\lambda}{(\lambda+\alpha_0)}(1-f^*(\lambda+\alpha_0)),$$

$$p_{49} = p_{46.9} = \frac{\alpha_0}{(\lambda+\alpha_0)}(1-r^*(\lambda+\alpha_0)),\ p_{37} = p_{5,10} = p_{78} = p_{74.8} = 1-g^*(\beta_0),$$

$$p_{14} = \frac{\beta_0}{(\lambda+\alpha_0+\beta_0)}(1-g^*(\lambda+\alpha_0+\beta_0)),\ p_{47} = \frac{\lambda}{(\lambda+\alpha_0)}(1-r^*(\lambda+\alpha_0)),$$

$$p_{31} = p_{56} = p_{74} = g^*(\beta_0),\ p_{26} = p_{84} = p_{96} = p_{10,6} = p_{11,1} = p_{12,6} = 1$$

It can be easily verify that

$$p_{01} + p_{02} = p_{10} + p_{13} + p_{14} + p_{15} = p_{26} = p_{40} + p_{47} + p_{49} = p_{60} + p_{6,11} + p_{6,12} = 1$$

$$p_{10} + p_{14} + p_{11.3} + p_{16.5} + p_{16.5,10} + p_{17.3} = p_{40} + p_{47} + p_{46.9} = 1$$

$$p_{60} + p_{61.11} + p_{66.12} = p_{74} + p_{74.8} = 1$$

The mean sojourn times (μ_i) is in the state S_i are

$\mu_0 = m_{01} + m_{02}$, $\mu_1 = m_{10} + m_{13} + m_{14} + m_{15}$, $\mu_2 = m_{26}$, $\mu_4 = m_{40} + m_{47} + m_{49}$, $\mu_6 = m_{60} + m_{6,11} + m_{6,12}$,

$\mu_1^{'} = m_{10} + m_{14} + m_{11.3} + m_{16.5} + m_{16.5,10} + m_{17.3}$,

$\mu_4^{'} = m_{40} + m_{47} + m_{46.9}$, $\mu_7 = m_{74} + m_{74.8}$,

Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state Si to a failed state. Regarding thefailed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_0(t) = Q_{01}(t) \& \phi_1(t) + Q_{02}(t)$$
$$\phi_1(t) = Q_{10}(t) \& \phi_0(t) + Q_{14}(t) \& \phi_4(t) + Q_{13}(t) + Q_{15}(t)$$
$$\phi_4(t) = Q_{40}(t) \& \phi_0(t) + Q_{48}(t) + Q_{49}(t) \tag{2}$$

Taking LST of above relation (7.4) and solving for $\Phi_0^{**}(s)$, we have

$$R^*(s) = \frac{1-\phi^{**}(s)}{s} \tag{3}$$

The reliability of the system model can be obtained by taking Inverse Laplace transform of (3). The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s\to 0} \frac{1-\phi^{**}(s)}{s} = \frac{N}{D} \tag{4}$$

Where

$$N = \mu_0 + p_{01}\mu_1 + p_{01}p_{14}\mu_4 \text{ and } D = 1 - p_{01}p_{10} - p_{01}p_{14}p_{40} \tag{5}$$

Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that thesystementered regenerative state Si at t = 0.The recursive relations for $A_i(t)$ are given as:

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$$
$$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + q_{11.3}(t) © A_1(t) + q_{14}(t) © A_4(t)$$
$$+ q_{17.3}(t) © A_7(t) + (q_{16.5}(t) + q_{16.5,10}(t)) © A_6(t)$$
$$A_2(t) = q_{26}(t) © A_6(t)$$
$$A_4(t) = M_4(t) + q_{40}(t) © A_0(t) + q_{47}(t) © A_7(t) + q_{46.9}(t) © A_6(t)$$
$$A_6(t) = M_6(t) + q_{60}(t) © A_0(t) + q_{61.11}(t) © A_1(t) + q_{66.12}(t) © A_6(t)$$
$$A_7(t) = (q_{74}(t) + q_{74.8}(t)) © A_4(t) \tag{6}$$

Where

$$M_0(t) = e^{-(2\lambda+\alpha_0)t},\ M_1(t) = e^{-(\lambda+\alpha_0+\beta_0)t}\overline{G(t)},$$
$$M_4(t) = e^{-(\lambda+\alpha_0)t}\overline{R(t)},\ M_6(t) = e^{-(\lambda+\alpha_0)t}\overline{F(t)} \tag{7}$$

Taking LT of above relation (6) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1} \tag{8}$$

Where

$$\begin{aligned} N_1 = {} & \mu_0[(1-p_{47})\{p_{60}(1-p_{11.3}) + p_{61.11}p_{10}\} + p_{61.11}p_{40}(p_{14}+p_{17.3})] \\ & + \mu_6[(1-p_{47})\{p_{01}(p_{16.5}+p_{16.5,10}) + p_{02}(1-p_{11.3})\} \\ & + p_{01}p_{46.9}(p_{14}+p_{17.3})] + \mu_1(1-p_{47})\{p_{01}(1-p_{66.12}) - p_{02}p_{61.11}\} \\ & + \mu_4(p_{14}+p_{17.3})\{p_{01}p_{60} + p_{61.11}\} \end{aligned} \tag{9}$$

$$
\begin{aligned}
D_1 = {} & (\mu_0 + \mu_2 p_{02})[(1-p_{47})\{p_{60}(1-p_{11.3}) + p_{61.11}p_{10}\} + p_{61.11}p_{40}(p_{14}+p_{17.3})] \\
& + \mu_6^{'}[(1-p_{47})\{p_{01}(p_{16.5}+p_{16.5,10}) + p_{02}(1-p_{11.3})\} + p_{01}p_{46.9}(p_{14}+p_{17.3})] \\
& + (p_{01}p_{60}+p_{61.11})\{\mu_4^{'}(p_{14}+p_{17.3}) + \mu_7^{'}(p_{14}p_{47}+p_{17.3})\} \\
& + \mu_1^{'}(1-p_{47})\{p_{01}(1-p_{66.12}) - p_{02}p_{61.11}\}
\end{aligned} \tag{10}
$$

Busy Period Analysis for Server

A. Due to Repair

Let $B_i^R(t)$ be the probability that the server is busy in repairof the unit at an instant't' given that the system entered regenerative state Si at t=0.The recursive relations for $B_i^R(t)$ are as follows:

$$
\begin{aligned}
& B_0^R(t) = q_{01}(t)©B_1^R(t) + q_{02}(t)©B_2^R(t) \\
& B_1^R(t) = W_1(t) + q_{10}(t)©B_0^R(t) + q_{11.3}(t)©B_1^R(t) + q_{14}(t)©B_4^R(t) \\
& \qquad + q_{17.3}(t)©B_7^R(t) + (q_{16.5}(t) + q_{16.5,10}(t))©B_6^R(t) \\
& B_2^R(t) = q_{26}(t)©B_6^R(t) \\
& B_4^R(t) = q_{40}(t)©B_0^R(t) + q_{47}(t)©B_7^R(t) + q_{46.9}(t)©B_6^R(t) \\
& B_6^R(t) = q_{60}(t)©B_0^R(t) + q_{61.11}(t)©B_1^R(t) + q_{66.12}(t)©B_6^R(t) \\
& B_7^R(t) = W_7(t) + (q_{74}(t) + q_{74.8}(t))©B_4^R(t)
\end{aligned} \tag{11}
$$

Where,

$$W_1(t) = e^{-(\lambda+\alpha_0+\beta_0)t}\overline{G(t)} + (\lambda e^{-(\lambda+\alpha_0+\beta_0)t}©1)\overline{G(t)} + (\alpha_0 e^{-(\lambda+\alpha_0+\beta_0)t}©1)\overline{G(t)}$$

and $W_7(t) = e^{-\beta_0 t}\overline{G(t)}$ (12)

Taking LT of above relation (11) and solving for $B_0^{R*}(s)$ .The time for which serveris busydue to repair is given by

$$B_0^R(\infty) = \lim_{s\to 0} sB_0^{R*}(s) = \frac{N_2}{D_1} \tag{13}$$

Where ,

$$N_2 = W_1^*(0)(1-p_{47})\{p_{01}(1-p_{66.12}) - p_{02}p_{61.11}\} + W_7^*(0)(p_{14}p_{47}+p_{17.3})\{p_{01}p_{60}+p_{61.11}\}$$

and D_1 is already mentioned. (14)

B. Due to Replacement

Let $B_i^{Rp}(t)$ be the probability that the server is busy in replacement of the unit at an instant 't' given that the system entered regenerative state Si at t=0.The recursive relations for $B_i^{Rp}(t)$ are as follows:

$$
\begin{aligned}
& B_0^{Rp}(t) = q_{01}(t)©B_1^{Rp}(t) + q_{02}(t)©B_2^{Rp}(t) \\
& B_1^{Rp}(t) = q_{10}(t)©B_0^{Rp}(t) + q_{11.3}(t)©B_1^{Rp}(t) + q_{14}(t)©B_4^{Rp}(t) \\
& \qquad + q_{17.3}(t)©B_7^{Rp}(t) + (q_{16.5}(t) + q_{16.5,10}(t))©B_6^{Rp}(t) \\
& B_2^{Rp}(t) = q_{26}(t)©B_6^{Rp}(t) \\
& B_4^{Rp}(t) = W_4(t) + q_{40}(t)©B_0^{Rp}(t) + q_{47}(t)©B_7^{Rp}(t) + q_{46.9}(t)©B_6^{Rp}(t) \\
& B_6^{Rp}(t) = q_{60}(t)©B_0^{Rp}(t) + q_{61.11}(t)©B_1^{Rp}(t) + q_{66.12}(t)©B_6^{Rp}(t) \\
& B_7^{Rp}(t) = (q_{74}(t) + q_{74.8}(t))©B_4^{Rp}(t)
\end{aligned} \tag{15}
$$

Where,

$$W_4(t) = e^{-(\lambda+\alpha_0)t}\overline{R(t)} + (\alpha_0 e^{-(\lambda+\alpha_0)t} © 1)\overline{R(t)} \quad (16)$$

Taking LT of above relation (15) and solving for $B_0^{Rp*}(s)$.The time for which server is busy due to replacement is given by

$$B_0^{Rp}(\infty) = \lim_{s\to 0} sB_0^{Rp*}(s) = \frac{N_3}{D_1} \quad (17)$$

Where,

$N_3 = W_4^*(0)(p_{14} + p_{17.3})(p_{01}p_{60} + p_{61.11})$ and D$_1$ is already mentioned. (18)

C. Due to Preventive Maintenance

Let $B_i^P(t)$ be the probability that the server is busy in preventive maintenance of the unit at an instant'‘t’ given that the system entered regenerative state Si at t=0.The recursive relations for $B_i^P(t)$ are as follows:

$$B_0^P(t) = q_{01}(t)©B_1^P(t) + q_{02}(t)©B_2^P(t)$$
$$B_1^P(t) = q_{10}(t)©B_0^P(t) + q_{11.3}(t)©B_1^P(t) + q_{14}(t)©B_4^P(t) + q_{17.3}(t)©B_7^P(t) + (q_{16.5}(t) + q_{16.5,10}(t))©B_6^P(t)$$
$$B_2^P(t) = W_2(t) + q_{26}(t)©B_6^P(t)$$
$$B_4^P(t) = q_{40}(t)©B_0^P(t) + q_{47}(t)©B_7^P(t) + q_{46.9}(t)©B_6^P(t)$$
$$B_6^P(t) = W_6(t) + q_{60}(t)©B_0^P(t) + q_{61.11}(t)©B_1^P(t) + q_{66.12}(t)©B_6^P(t)$$
$$B_7^P(t) = (q_{74}(t) + q_{74.8}(t))©B_4^P(t) \quad (19)$$

Where,

$$W_6(t) = e^{-(\lambda+\alpha_0)t}\overline{F(t)} + (\alpha_0 e^{-(\lambda+\alpha_0)t}©1)\overline{F(t)} + (\lambda e^{-(\lambda+\alpha_0)t}©1)\overline{F(t)} \text{ and } W_2(t) = \overline{F(t)} \quad (20)$$

Taking LT of above relation (19) and solving for $B_0^{P*}(s)$.The time for which server is busy due to preventive maintenance is given by

$$B_0^P(\infty) = \lim_{s\to 0} sB_0^{P*}(s) = \frac{N_4}{D_1} \quad (21)$$

Where,

$$N_4 = W_2^*(0)p_{02}[(1-p_{47})\{p_{60}(1-p_{11.3}) + p_{61.11}p_{10}\}p_{61.11}p_{40}(p_{14}+p_{17.3})] + W_6^*(0)[(1-p_{47})\{p_{01}(p_{16.5}+p_{16.5,10}) + p_{02}(1-p_{11.3})\} + p_{01}p_{46.9}(p_{14}+p_{17.3})]$$

and D$_1$ is already mentioned. (22)

Expected Number of Repairs

Let $R_i(t)$ be the expected number of repairs by the server in (0, t] given that the system entered the regenerative state Si at t = 0. The recursive relations for $R_i(t)$ are given as:

$$R_0(t) = Q_{01}(t)\&R_1(t) + Q_{02}(t)\&R_2(t)$$
$$R_1(t) = Q_{10}(t)\&[1+R_0(t)] + Q_{11.3}(t)\&[1+R_1(t)] + Q_{14}(t)\&R_4(t) + Q_{17.3}(t)\&R_7(t) + Q_{16.5}(t)\&[1+R_6(t)] + Q_{16.5,10}(t)\&R_6(t)$$
$$R_2(t) = Q_{26}(t)\&R_6(t)$$
$$R_4(t) = Q_{40}(t)\&R_0(t) + Q_{47}(t)\&R_7(t) + Q_{46.9}(t)\&R_6(t)$$
$$R_6(t) = Q_{60}(t)\&R_0(t) + Q_{61.11}(t)\&R_1(t) + Q_{66.12}(t)\&R_6(t)$$
$$R_7(t) = Q_{74}(t)\&[1+R_4(t)] + Q_{74.8}(t)\&R_4(t) \quad (23)$$

Taking LST of above relations (23) and solving for $R_0^{**}(s)$.The expected no. of repairs per unit time by the server are giving by

$$R_0(\infty) = \lim_{s\to 0} sR_0^{**}(s) = \frac{N_5}{D_1} \tag{24}$$

Where,

$$N_5 = (p_{10} + p_{11.3} + p_{16.5})(1 - p_{47})\{p_{01}(1 - p_{66.12}) - p_{02}p_{61.11}\}$$
$$+ p_{74}(p_{14}p_{47} + p_{17.3})(p_{01}p_{60} + p_{61.11})$$

and D_1 is already mentioned. (25)

Expected Number of Replacements

Let $Rp_i(t)$ be the expected number of replacements by the server in (0, t] given that the system entered the regenerative state Si at t = 0. The recursive relations for $Rp_i(t)$ are given as:

$$Rp_0(t) = Q_{01}(t)\& Rp_1(t) + Q_{02}(t)\& Rp_2(t)$$
$$Rp_1(t) = Q_{10}(t)\& Rp_0(t) + Q_{11.3}(t)\& Rp_1(t) + Q_{14}(t)\& Rp_4(t)$$
$$+ Q_{17.3}(t)\& Rp_7(t) + Q_{16.5,10}(t)\& [1 + Rp_6(t)] + Q_{16.5}(t)\& Rp_6(t)$$
$$Rp_2(t) = Q_{26}(t)\& Rp_6(t)$$
$$Rp_4(t) = Q_{40}(t)\& [1 + Rp_0(t)] + Q_{47}(t)\& Rp_7(t) + Q_{46.9}(t)\& [1 + Rp_6(t)]$$
$$Rp_6(t) = Q_{60}(t)\& Rp_0(t) + Q_{61.11}(t)\& Rp_1(t) + Q_{66.12}(t)\& Rp_6(t)$$
$$Rp_7(t) = Q_{74}(t)\& Rp_4(t)] + Q_{74.8}(t)\& [1 + Rp_4(t)] \tag{26}$$

Taking LST of above relations (26) and solving for $Rp_0^{**}(s)$.The expected no. of replacements per unit time by the server are giving by

$$Rp_0(\infty) = \lim_{s\to 0} sRp_0^{**}(s) = \frac{N_6}{D_1} \tag{27}$$

Where,

$$N_6 = p_{16.5,10}(1 - p_{47})\{p_{01}(1 - p_{66.12}) - p_{02}p_{61.11}\}$$
$$+ (p_{40} + p46.9p)(p_{14} + p_{17.3})\{p_{01}p_{60} + p_{61.11}\}$$
$$+ p_{74.8}(p_{14}p_{47} + p_{17.3})(p_{01}p_{60} + p_{61.11})$$

and D_1 is already mentioned. (28)

Expected Number of Preventive Maintenances

Let $P_i(t)$ be the expected number of preventive maintenance by the server in (0, t] given that the system entered the regenerative state $S_i$at t = 0. The recursive relations for $P_i(t)$ are given as:

$$P_0(t) = Q_{01}(t)\& P_1(t) + Q_{02}(t)\& P_2(t)$$
$$P_1(t) = Q_{10}(t)\& P_0(t) + Q_{11.3}(t)\& P_1(t) + Q_{14}(t)\& P_4(t)$$
$$+ Q_{17.3}(t)\& P_7(t) + \{Q_{16.5}(t) + Q_{16.5,10}(t)\}\& P_6(t)$$
$$P_2(t) = Q_{26}(t)\& [1 + P_6(t)]$$
$$P_4(t) = Q_{40}(t)\& P_0(t) + Q_{47}(t)\& P_7(t) + Q_{46.9}(t)\& P_6(t)$$
$$P_6(t) = Q_{60}(t)\& [1 + P_0(t)] + Q_{61.11}(t)\& [1 + P_1(t)] + Q_{66.12}(t)\& [1 + P_6(t)]$$
$$P_7(t) = \{Q_{74}(t) + Q_{74.8}(t)\}\& P_4(t) \tag{29}$$

Taking LST of above relations (29) and solving for $P_0^{**}(s)$.The expected no. of preventive maintenances per unit time by the server are giving by

$$P_0(\infty) = \lim_{s\to 0} sP_0^{**}(s) = \frac{N_7}{D_1} \tag{30}$$

Where,

$$N_7 = p_{02}[(1-p_{47})\{p_{60}(1-p_{11.3}) + p_{61.11}p_{10}\} + p_{61.11}p_{40}(p_{14}+p_{17.3})]$$
$$+[(1-p_{47})\{p_{01}(p_{16.5}+p_{16.5,10}) + p_{02}(1-p_{11.3})\} + p_{01}p_{46.9}(p_{14}+p_{17.3})]$$

and D_1 is already mentioned. (31)

Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^R - K_2B_0^{Rp} - K_3B_0^P - K_4R_1 - K_5Rp_0 - K_6P_0 \tag{32}$$

Where,

P = Profit of the system model after reducing cost per unit time busy of the server and cost per repair activity per unit time

K_0 = Revenue per unit up-time of the system

K_1 =Cost per unit time for which server is busy due to repair

K_2 =Cost per unit time for which server is busy due to replacement

K_3 =Cost per unit time for which server is busy due to preventive maintenance

K_4 = Cost per repair per unit time

K_5 = Cost per replacement per unit time

K_6 = Cost per preventive maintenance per unit time

CONCLUSION

After solving the equations of MTSF, availability and profit function for the particular case g(t) = $\theta e^{-\theta t}$, r(t) = $\beta e^{-\beta t}$ and f(t) = $\alpha e^{-\alpha t}$, we conclude that

- These reliability measures are increasing as the repair rate θ, replacement rate β increases while their values decline with the increase of failure rate λ and the rate α_0 by which the unit undergoes for preventive maintenance.
- The MTSF and availability keep on upwards with the increase of the rate β_0 by which unit undergoes for replacement but profit decreases.
- The system model becomes more profitable when we increase the preventive maintenance rate α.

GRAPHS FOR PARTICULAR CASE

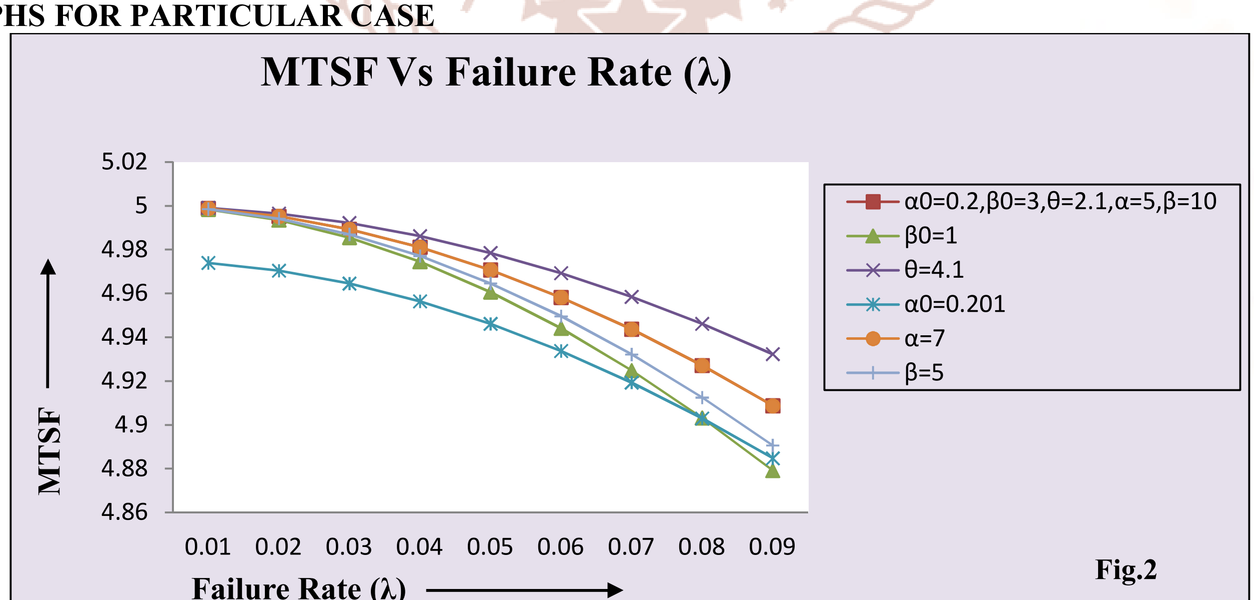

Fig.2

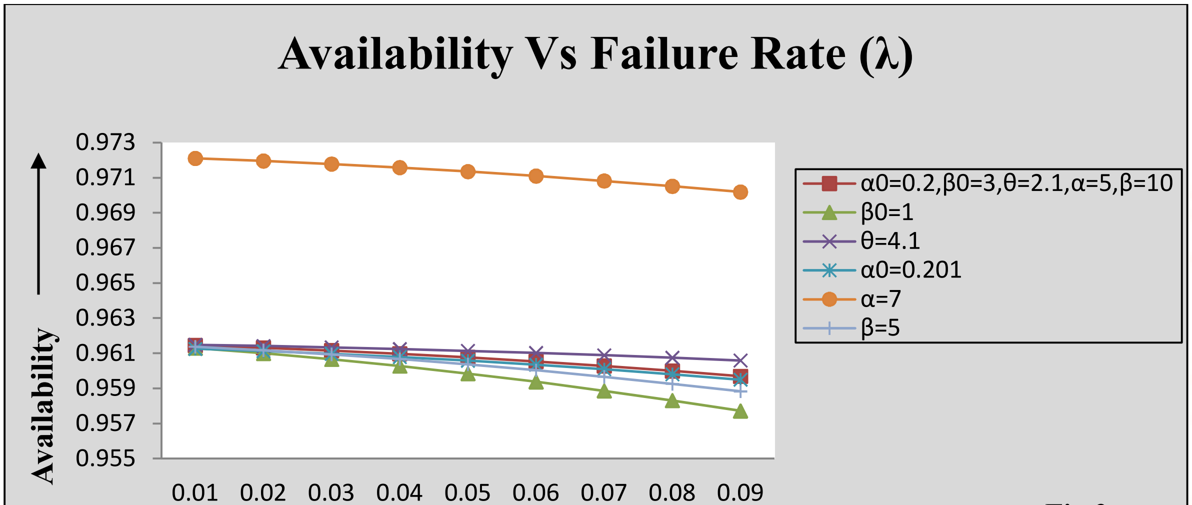

Fig.3

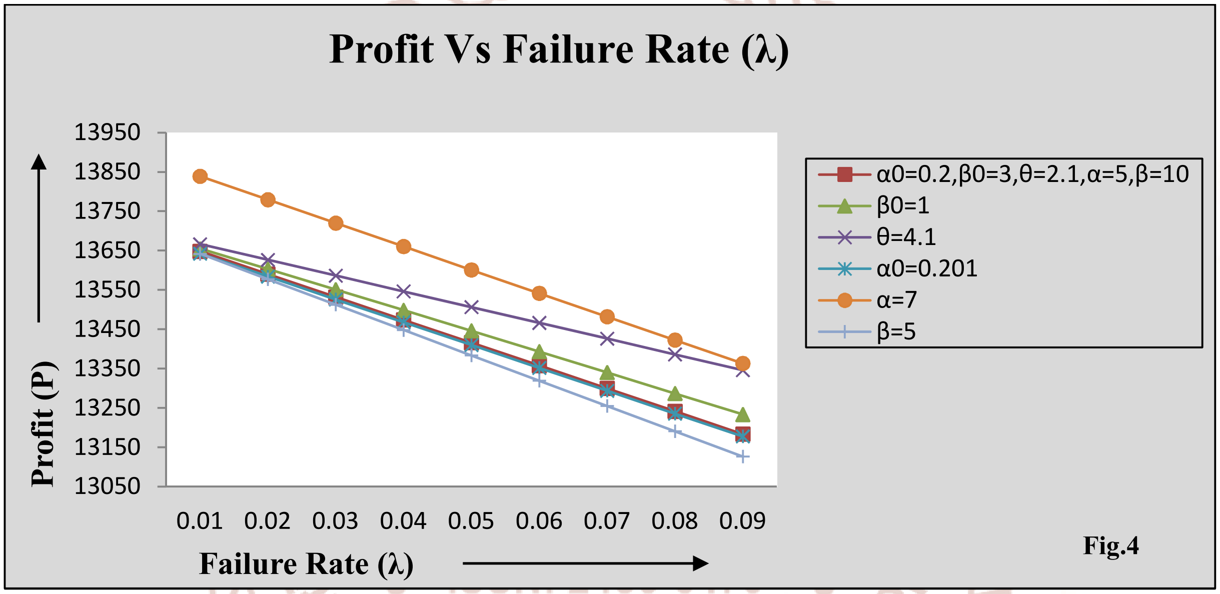

Fig.4

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