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Integration of Measurable Functions with Complex Values with Respect to a Positive Measure

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ABSTRACT

This article presents a comprehensive study of integration theory for complex-valued measurable functions with respect to a positive measure. The main objective is to establish the necessary and sufficient conditions for the integrability of complex-valued measurable functions and to determine the algebraic structure of the set of these integrable functions. We prove that a measurable function $f: X \to \mathbb{C}$is integrable on $\mathbb{R}$ $E \in \mathcal{A}$if and only if both its real $\mathrm{Re}(f)$and imaginary parts $\mathrm{Im}(f)$are integrable on $\mathbb{R}$ E. Furthermore, we prove that the set $\mathcal{L}(X, \mathcal{A}, \mu, \mathbb{C})$of complex-valued integrable functions forms a vector space on $\mathbb{R}$ $\mathbb{C}$equipped with the usual operations of addition and scalar multiplication. Several fundamental properties are established, including linearity, the relationship between the integrability of $\mathbb{R}$ $f$and that of $\mathbb{R}$ $|f|$, and the behavior of the integral $\mathbb{R}$ on sets of measure zero. This work extends the classical results of real integration theory to the complex setting and provides a rigorous foundation for applications in analysis, probability theory, and mathematical physics.

KEYWORDS: *Measurable functions, complex-valued functions, positive measure, Lebesgue integral, vector space, integrability conditions, real and imaginary parts, measure theory.*

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1. INTRODUCTION

The theory of integration constitutes one of the pillars fundamentals of analysis mathematical modern. Since the pioneering work Introduced by Henri Lebesgue at the beginning of the XX^ecentury, the Lebesgue integral revolutionized our understanding of functions and measures, offering a much more general and powerful framework than the classical Riemann integral [1], [2]. The Lebesgue integral not only allows the integration of a much broader class of functions, but it also provides extremely robust convergence theorems that are essential in functional analysis, probability theory, and mathematical physics.

Measure theory, developed concurrently by mathematicians such as Borel, Jordan, and Lebesgue, provides the foundation upon which modern integration rests [3], [4]. A measure is a process that associates to each set $A$of a given sigma-algebra a positive number $\mu(A)$, called the measure of the function A, satisfying properties of monotonicity and countable additivity. The Lebesgue integral of a function is then constructed from this measure, by generalizing the notion of the area under the curve.

While the theory of integration of real-valued functions is now well established in the mathematical literature, its natural extension to complex-valued functions deserves special attention. Indeed, many applications in complex analysis, quantum physics, signal processing, and probability require the integration of complex functions [5], [6]. Complex-valued functions naturally arise as linear combinations of real-valued functions via decomposition $f = \mathrm{Re}(f) + i\mathrm{Im}(f)$.

In integration theory, the Lebesgue integral is well established for positive real functions. However, when this notion is extended to measurable functions with complex values, several fundamental questions arise:

- How can we rigorously define the integral of a measurable function with complex values?
- What conditions guarantee the existence of this integral?
- How can we link integration in the Riemannian sense to integration in the Lebesgue sense?
- Which properties of the real integral (linearity, convergence, etc.) remain valid in the complex case?

Therefore, in order to better define the scope of the problem in our study, we have arranged the following questions:

- How can Lebesgue integration be generalized to measurable functions with complex values defined on a space equipped with a positive measure, while guaranteeing the consistency, existence and fundamental properties of the integral?

To address this issue, we can formulate the following hypotheses:

- Any integrable complex-valued measurable function can be decomposed into two measurable functions (real part and imaginary part), whose integrability guarantees that of the complex function. The complex-valued measurable function $f$is written as:

$$f = Re(f) + Im(f)$$

Where $\mathcal{Re}(\mathcal{F})\ et\ im(\mathcal{F})$are measurable

For the development of this research, we used the following methods:

Analytical method, comparative method, approximation method, standard deviation method, analytical method and generalization method

These methods were complemented by the following techniques:

Use of measurable functions, Separate integration, Convergence techniques and bounding and inequalities.

The objective of this work is threefold:

1. Establish the necessary and sufficient conditions for the integrability of measurable functions with complex values relative to a positive measure.
2. To rigorously demonstrate the fundamental properties of the complex integral, in particular the linearity and the relationship between the integrability of $f$and that of $|f|$.
3. Determine the algebraic structure of the set $\mathcal{L}(X, \mathcal{A}, \mu, \mathbb{C})$of integrable complex functions.

This work is a continuation of the research conduct in Theory of measure and integration [7, 8, 9, 10]. It complements existing work in providing demonstrations detailed properties often stated without proof complete in the literature.

2. THEORETICAL APPROACH

2.1. Measurable Spaces and Measures [3, 4, 7, 8, 9, 10, 11]

2.1.1. Definition of an σalgebra

Either Xa non-empty set. A σσ-algebra (or sigma-algebra) on is Xa family that satisfies the $\mathcal{A} \subset \mathcal{P}(X)$following axioms :

1. $\varnothing \in \mathcal{A}$;
2. If $A \in \mathcal{A}$, then $X \backslash A \in \mathcal{A}($ $\mathcal{A}$is stable under the complement);
3. If $(A_n)_{n\geq 1}$is a sequence of elements of $\mathcal{A}$, then $\bigcup_{n=1}^{\infty} A_n \in \mathcal{A}($ $\mathcal{A}$is closed under countable union).

The elements of $\mathcal{A}$are called the measurable parts of X. We say that $(X, \mathcal{A})$is a measurable space.

2.1.2. Proposal

If $(X, \mathcal{A})$is a measurable space, then the following properties are verified:

1. $X \in \mathcal{A}$;
2. If $(B_n)_{n\geq 1}$is a sequence of elements of $\mathcal{A}$, then $\bigcap_{n=1}^{\infty} B_n \in \mathcal{A}$;
3. If $p \in \mathbb{N}$and $A_1, A_2, \ldots, A_p \in \mathcal{A}$, then $\bigcap_{k=1}^{p} A_k \in \mathcal{A}$and $\bigcup_{k=1}^{p} A_k \in \mathcal{A}$;
4. If $A, B \in \mathcal{A}$, then $A \backslash B \in \mathcal{A}$and $A \triangle B \in \mathcal{A}$.

Evidence 1.

1. Since $\varnothing \in \mathcal{A}$, by property (ii), $X = X \backslash \varnothing \in \mathcal{A}$.
2. Let be $(B_n)_{n\geq 1}$a sequence in $\mathcal{A}$. By property (ii), $X \backslash B_n \in \mathcal{A}$for all n. By property (iii), $\bigcup_{n=1}^{\infty}(X \backslash B_n) \in \mathcal{A}$. Thus,
$$\bigcap_{n=1}^{\infty} B_n = X \backslash \bigcup_{n=1}^{\infty}(X \backslash B_n) \in \mathcal{A}.$$
3. Deduces from properties (ii) and (iii) by induction.
4. $A \backslash B = A \cap (X \backslash B) \in \mathcal{A}$ according to property (iii) (finite intersection). Furthermore, $A \triangle B = (A \backslash B) \cup (B \backslash A) \in \mathcal{A}$.

2.1.3. Borelian Tribe

Definition 2.1. Let Xany set. If $\mathcal{B}$is a family of subsets of X(that is, $\mathcal{B} \subset \mathcal{P}(X)$), then the family

$$\mathcal{A} \equiv \bigcap\{\mathcal{S}: \mathcal{S} \text{ est une } \sigma\text{-algèbre sur } X \text{ avec } \mathcal{B} \subset \mathcal{S}\}$$

East a σ-algebra on satisfying the Xfollowing properties :

1. $\mathcal{B} \subset \mathcal{A}$;
2. For any σ-algebra $\mathcal{G}$on Xsuch that $\mathcal{B} \subset \mathcal{G}$, we have $\mathcal{A} \subset \mathcal{G}$.

$\mathcal{A}$is called the σ-algebra generated by $\mathcal{B}$.

Definition 2.2. Let be (X, τ)a topological space. The σ-algebra generated by τon $X$is called the σ-algebra of Borel sets of X, or the Borel sigma-algebra on X. It is denoted $\mathcal{B}(X)$.

2.1.4. Families that give rise to the Borelians of $\mathbb{R}$

The tribe Borelian $\mathcal{B}(\mathbb{R})$is the sigma-algebra generated by the class of open intervals. An element of this sigma-algebra is called a Borel subset, or a Borel.

Every open, closed, or half-open interval belongs to the set $\mathcal{B}(\mathbb{R})$of intervals. The same is true of any finite or countable union of intervals. Thus, we have the following generating sets:

$$\mathcal{B}_1 = \{]a, b[: a \le b \text{ dans } \mathbb{R}\};$$
$$\mathcal{B}_2 = \{[a, b]: a \le b \text{ dans } \mathbb{R}\};$$
$$\mathcal{B}_3 = \{]a, b]: a \le b \text{ dans } \mathbb{R}\};$$
$$\mathcal{B}_4 = \{[a, b[: a \le b \text{ dans } \mathbb{R}\};$$
$$\mathcal{B}_5 = \{]-\infty, a[: a \in \mathbb{R}\};$$
$$\mathcal{B}_6 = \{]-\infty, a]: a \in \mathbb{R}\};$$
$$\mathcal{B}_7 = \{[a, +\infty[: a \in \mathbb{R}\};$$
$$\mathcal{B}_8 = \{]a, +\infty[: a \in \mathbb{R}\};$$
$$\mathcal{B}_9 = \{I \subset \mathbb{R}: I \text{ est un intervalle}\}.$$

2.1.5. Positive measure

Definition 2.3. Either a measurable space. A $(X, \mathcal{A})$*positive measure* on $(X, \mathcal{A})$a function satisfying the $\mu: \mathcal{A} \to [0, +\infty]$following conditions is called :

1. $\mu(\varnothing) = 0$;
2. *Countable additivity* : If $(A_n)_{n\ge1}$is a sequence of pairwise disjoint elements of $\mathcal{A}$, then

$$\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n).$$

A *space measure* is a triplet $(X, \mathcal{A}, \mu)$where $\mathcal{A}$is an σ-algebra on $X$and $\mu$is a positive measure on $\mathcal{A}$.

2.1.6. Properties of a positive measure

Either $(X, \mathcal{A}, \mu)$A measured space. The measure has the μfollowing properties:

1. *Finite additivity* **:** If $n \in \mathbb{N}$and $A_1, \ldots, A_n$are pairwise disjoint, then $\mu(\bigcup_{k=1}^n A_k) = \sum_{k=1}^n \mu(A_k)$.
2. *Monotony:* If $A \subset B$, then $\mu(A) \le \mu(B)$; if in addition $\mu(B) < +\infty$, then $\mu(B \backslash A) = \mu(B) - \mu(A)$.
3. *Countable subadditivity:* For any sequence (D_n)in $\mathcal{A}$, $\mu(\bigcup_{n=1}^\infty D_n) \le \sum_{n=1}^\infty \mu(D_n)$.
4. *Finite subadditivity* **:** $\mu(\bigcup_{k=1}^n A_k) \le \sum_{k=1}^n \mu(A_k)$.
5. *Monotonic convergence (left continuity): If* $(A_n)$is increasing, $\mu(\bigcup_{n=1}^\infty A_n) = \lim_{n\to+\infty} \mu(A_n)$.
6. *Decreasing convergence (right continuity):* If $(A_n)$is decreasing with $\mu(A_1) < +\infty$, then $\mu(\bigcap_{n=1}^\infty A_n) = \lim_{n\to+\infty} \mu(A_n)$.

2.1.7. A negligible part and "almost everywhere"

Either $(X, \mathcal{A}, \mu)$a measured space. A part $E \subset X$is said to μ*be negligible.* if exists $A \in \mathcal{A}$such that $\mu(A) = 0$and $E \subset A$. A property $\mathcal{P}$is said to be true μ*-almost everywhere* (μ-pp) if the complement of the set of points where it is realized is μ-negligible.

Let f $f, g: X \to \mathbb{F}$(x $\mathbb{F} = \mathbb{R}$), f(x), $\overline{\mathbb{R}}$or f(x) be $\mathbb{C}$two measurable functions. We say that they are *equal* μ*— almost everywhere—* when f $\{x: f(x) \neq g(x)\}$is negligible. We denote f(x) by f(x).$f = g$ μ-pp.

2.1.8. Lebesgue measurement

The *Lebesgue measure* on $\mathbb{R}$is the unique measure μdefined on the Borel sigma-algebra $\mathcal{B}(\mathbb{R})$such that for any $[a, b]$bounded interval, $\mu([a, b]) = b - a$. The Lebesgue measure is complete and invariant under translation.

2.1.9. Measurable Applications [2, 5, 6, 8, 9, 11]

Definition 2.4. Let $(X, \mathcal{A})$and $(Y, \mathcal{S})$measurable spaces. An application $f: X \to Y$is said to be measurable. relative to $\mathcal{A}$**and** $\mathcal{S}$(or $\mathcal{A}$- $\mathcal{S}$measurable) if $\forall T \in \mathcal{S}, f^{-1}(T) \in \mathcal{A}$. We denote $M(X, Y) = \{f \in Y^X: f \text{ est mesurable}\}$.

Definition 2.5. Let be $(X, \mathcal{A})$a measurable space. A function $f: X \to \mathbb{R}$is measurable if, for all $\alpha \in \mathbb{R}$, the following sets are measurable:

$$f^{-1}(]\alpha, +\infty[), \quad f^{-1}([\alpha, +\infty[), \quad f^{-1}(]-\infty, \alpha]), \quad f^{-1}(]-\infty, \alpha[).$$

Theorem 2.1. (Measurability criterion for complex functions). *Consider a space measurable* $(X, \mathcal{A})$*and a complex function* $f: X \to \mathbb{C}$. *Then* f*is measurable if and only if the real part* $\mathrm{Re}(f)$*and the imaginary part* $\mathrm{Im}(f)$*of* f*are measurable.*

Proof 2. Let us define $g, h: \mathbb{C} \to \mathbb{R}$by $g(z) = \text{Re}(z)$and $h(z) = \text{Im}(z)$. These functions are continuous, therefore measurable.

Suppose it $f$is measurable. Then $g \circ f = \text{Re}(f)$and $h \circ f = \text{Im}(f)$are measurable.

Conversely $f = S \circ T$, suppose $\text{Re}(f)$that $\text{Im}(f)$\$ $T(x) = (\text{Re}(f(x)), \text{Im}(f(x)))$\ $S: \mathbb{R}^2 \to \mathbb{C}$frac { $S(u,v) = u + iv$...$T: X \to \mathbb{R}^2$

2.2. Integration of Real-Valued Functions [1, 2, 3, 7, 8, 9, 11, 12]

2.3.1. Positive stepped functions

Definition 2.6. A function $f: X \to \overline{\mathbb{R}}$is said *to be tiered* if She East measurable and does not take that a number finite of values. We denote $\mathcal{E}^+$by the set of positive step functions (with values in $[0, +\infty]$). Its canonical representation is

$$f = \sum_{k=1}^{n} c_k \mathbf{1}_{A_k},$$

Or $c_k \in [0, +\infty]$, $A_k \in \mathcal{A}$two by two disjoint, and $\bigcup_{k=1}^{n} A_k = X$.

The integral of $f \in \mathcal{E}^+$over $E \in \mathcal{A}$is defined by

$$\int_E f \, d\mu = \sum_{k=1}^{n} c_k \, \mu(A_k \cap E).$$

2.3.2. Positive measurable functions

For $f \in M^+$, the integral of $f$on $E \in \mathcal{A}$is

$$\int_E f \, d\mu = \sup\left\{\int_E u \, d\mu : u \in \mathcal{E}^+, u \leq f\right\}.$$

2.3.3. Integrable real functions

A function $f: X \to \mathbb{R}$is said *$\mu$to be -integrable* on $E \in \mathcal{A}$if $f^+$and $f^-$are finite integrals on E. In this case,

$$\int_E f \, d\mu = \int_E f^+ \, d\mu - \int_E f^- \, d\mu.$$

2.3.4. Integration of Complex Value Functions [5, 6, 7, 9, 10, 13]

Definition 2.7. Either $(X, \mathcal{A}, \mu, \mathbb{C})$a measure space and $E \in \mathcal{A}$. A function $f: X \to \mathbb{C}$is said **$\mu$to be -integrable** on Eif $\text{Re}(f)$and $\text{Im}(f)$are both μ-integrable on E. In this case,

$$\int_E f \, d\mu = \int_E \text{Re}\,(f) \, d\mu + i \int_E \text{Im}\,(f) \, d\mu.$$

We denote $\mathcal{L}(X, \mathcal{A}, \mu, \mathbb{C})$by the set of functions μ-integrable on $X$with complex values.

3. RESULTS

Theorem 3.1. (Criterion integrability). *That is $f: X \to \mathbb{C}$a measurable function and $E \in \mathcal{A}$. Then $f$is μ-integrable on Eif and only if $|f|$is μ-integrable on E.*

Proof 3. Suppose$f$ μ-integrable on E. So $\text{Re}(f)$and $\text{Im}(f)$are, therefore $|\text{Re}(f)|$and $|\text{Im}(f)|$are also. Since $|f| \leq |\text{Re}(f)| + |\text{Im}(f)|$, we have

$$\int_E |f| \, d\mu \leq \int_E |\text{Re}(f)| \, d\mu + \int_E |\text{Im}(f)| \, d\mu < +\infty.$$

SO $|f|$is μ-integrable.

Suppose$|f|$ μ-integrable on E. Then $\int_E |f| \, d\mu < +\infty$. Since $|\text{Re}(f)| \leq |f|$and $|\text{Im}(f)| \leq |f|$, we obtain the integrability of $\text{Re}(f)$and $\text{Im}(f)$, therefore of f.

Theorem 3.2. (Linearity). *Let $f, g: X \to \mathbb{C}$two functions μbe integrable on $E \in \mathcal{A}$and $c \in \mathbb{C}$. Then:*

1. *$f + g$ East μ-integrable on E, and $\int_E (f + g) \, d\mu = \int_E f \, d\mu + \int_E g \, d\mu$.*
2. *cf East μ-integrable on E, and $\int_E c\, f \, d\mu = c \int_E f \, d\mu$.*

Evidence 4.

1. We have $\text{Re}(f + g) = \text{Re}(f) + \text{Re}(g)$and $\text{Im}(f + g) = \text{Im}(f) + \text{Im}(g)$. The real and imaginary parts are integrable, therefore $f + g$is. Direct calculus gives the additivity.
2. Let $c = \alpha + i\beta$. Then $\text{Re}(cf) = \alpha\text{Re}(f) - \beta\text{Im}(f)$and $\text{Im}(cf) = \alpha\text{Im}(f) + \beta\text{Re}(f)$are integrable; hence $cf$is integrable. An explicit calculation gives the formula.

Corollary 3.3. *The set $\mathcal{L}(X, \mathcal{A}, \mu, \mathbb{C})$is a vector space over $\mathbb{C}$equipped with addition and scalar multiplication.*

Theorem 3.4. (Triangle Inequality). *If $f: X \to \mathbb{C}$is μ-integrable on $E \in \mathcal{A}$, then$\left|\int_E f \, d\mu\right| \leq \int_E |f| \, d\mu$.*

Proof 5. Let $I = \int_E f \, d\mu$If so $I = 0$, it's trivial. Otherwise, let's write it $I = |I|e^{i\theta}$. Then

$$\begin{aligned} |I| = e^{-i\theta} \int_E f \, d\mu &= \int_E e^{-i\theta} f \, d\mu \\ &= \text{Re}\left(\int_E e^{-i\theta} f \, d\mu\right) \\ &= \int_E \text{Re}\,(e^{-i\theta} f) \, d\mu \leq \int_E |f| \, d\mu. \end{aligned}$$

Theorem 3.5. (Integrity and measure sets) null). *If $f: X \to \mathbb{C}$is measurable and $E \in \mathcal{A}$with $\mu(E) = 0$, then $f$is μ-integrable on $E$and $\int_E f \, d\mu = 0$.*

Theorem 3.6. (Equality almost everywhere). *If $f, g \in M(X, \mathbb{C})andf = g$ μ-pp, then for all $E \in \mathcal{A}$,*

f is μ-integrable on E if and only if g the is, and in that case $\int_E f\ d\mu = \int_E g\ d\mu$.

3.1. Algebraic Structure

Theorem 8. The *application* $\Phi: \mathcal{L}(X, \mathcal{A}, \mu, \mathbb{C}) \to \mathbb{C}$ *defined by* $\Phi(f) = \int_X f\ d\mu$ *is a linear form on the vector space* $\mathcal{L}(X, \mathcal{A}, \mu, \mathbb{C})$.

Note 1. The space $\mathcal{L}(X, \mathcal{A}, \mu, \mathbb{C})$ is isomorphic to

$$\mathcal{L}(X, \mathcal{A}, \mu, \mathbb{R}) \times \mathcal{L}(X, \mathcal{A}, \mu, \mathbb{R})$$

via $f \mapsto (\mathrm{Re}(f), \mathrm{Im}(f))$.

3.2. Additional Properties

Proposition 3.7. *Let $A, B \in \mathcal{A}$ disjoint and f measurable. If f is μ-integrable on $A \cup B$, then f is integrable on A and B, and $\int_{A\cup B} f\ d\mu = \int_A f\ d\mu + \int_B f\ d\mu$.*

Proposition 3.8. *If exists $g: X \to \mathbb{R}$ μ-integrable on E with $g \geq 0$ and $|f| \leq g$ on E, then f is μ-integrable on E and $\left|\int_E f\ d\mu\right| \leq \int_E |f|\, d\mu \leq \int_E g\ d\mu$.*

Proposition 11. If *and $\mu(E) < +\infty$ for $f(x) = c \in \mathbb{C}$ all $x \in E$, then f is μ-integrable on E and $\int_E f\ d\mu = c\,\mu(E)$.*

3.3. Dominated Convergence Theorem

Theorem 3.9. (Lebesgue dominated convergence). *Let (f_n) a sequence of integrable functions, $f_n: X \to \mathbb{C}$ and $f: X \to \mathbb{C}$ a function defined almost everywhere. We assume:*

$f_n(x) \to f(x)$ μ-pp;

There are $g: X \to \mathbb{R}^+$ integrable options such as $|f_n(x)| \leq g(x)$ μ-pp for everything n.

So f it is integrable and for everything $E \in \mathcal{A}$, $\int_E f\ d\mu = \lim_{n\to+\infty} \int_E f_n\ d\mu$.

4. CONCLUSION

In this work, we have developed a theory complete integration of functions measurable with complex values relative to a positive measure. Our main results are as follows :

1. A measurable function $f: X \to \mathbb{C}$ is μ-integrable on $E \in \mathcal{A}$ if and only if its real and imaginary parts are both μ-integrable on E.
2. The set $\mathcal{L}(X, \mathcal{A}, \mu, \mathbb{C})$ of integrable complex functions forms a vector space on $\mathbb{C}$, isomorphic to the product $\mathcal{L}(X, \mathcal{A}, \mu, \mathbb{R}) \times \mathcal{L}(X, \mathcal{A}, \mu, \mathbb{R})$.
3. The integral map $f \mapsto \int_E f\ d\mu$ is a linear form on this vector space.
4. The triangle inequality $\left|\int_E f\ d\mu\right| \leq \int_E |f|\, d\mu$ holds for any integrable complex function.
5. A measurable function f is integrable if and only if $|f|$ is integrable.
6. The integral of a function over a set of measure zero is zero.
7. Two functions that are equal almost everywhere have the same integrability and the same integral.

These results extend naturally the theory classic integration real and provide a rigorous framework for applications in analysis complex, in probability and mathematical physics.

Interesting perspectives include extending this theory to spaces $\mathcal{L}^p$ for complex-valued functions, as well as studying convergence theorems in this complex setting.

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