International Journal of Trend in Scientific Research and Development (IJTSRD)
International Open Access Journal
ISSN No: 2456 - 6470 | www.ijtsrd.com | Volume - 2 | Issue – 3

A Case Study in Application of Vectors in Three Dimensional Spaces

S. Sathyapriya[1], D. Harshini Devi[2], N. Priyanka[3]
[1]Assistant Professor
[1,2,3] Sri Krishna Arts and Science College, Coimbatore, Tamil Nadu, India

ABSTRACT

This paper presents a concept of vectors in three dimensional space and it will start by introducing the subject matter as well as giving a brief history on vectors in three dimensional space. This paper also gives different examples of vectors in three dimensional space and how they can be used to solve various real life problems. This concept have many applications in physics and engineering. For instance Vectors in space can be used to represent the physical force and velocity.

Keywords: Vectors in space, force, velocity, coordinate system, tension, equilibrium

Introduction

Analytical geometry can be defined as a branch of mathematics that is concerned with carrying out geometric investigations using various algebric procedures.

In space just as in the plane, the sets of equivalent directed line segments (or arrows) are vectors. Vectors play an important role in **physics:** the velocity and acceleration of a moving object and the forces acting on it can all be described with vectors. Many other physical quantities can be usefully thought of as vectors. Although most of them do not represent distances,(except for example position or displacement) their magnitude and direction can still be represented by the length and direction of an arrow. The mathematical representation of a physical vector depends on the coordinate system used to describe it. Other vector like objects that describe physical quantities and transform in a similar way under changes of the coordinate system used to describe it. Other vector like objects that describe physical quantities and transform in a similar way under changes of the coordinate system.

When the concept of vector is extended to three-dimensional space, they are denoted by ordered triples $v = \langle v_1,v_2,v_3\rangle$

Simply vector quantities can be expressed geometrically. However, as the application become more complex, or involve a third dimension, you will need to be able to express vectors in cartesian coordinates, that is x,y and z coordinates. In this chapter we will investigate this Cartesian vectors and develop a three dimensional coordinate system. We will also develop vector products and explore their applications.

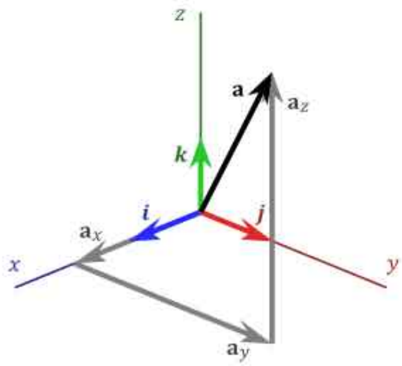
Vectors in two dimensions

On a two dimensional diagram, sometimes a vector perpendicular to the plane of the diagram is desired. These vectors are commonly shown in small circles. A circle with a dot at its centre indicates a vector pointing out of the front of the diagram, toward the viewer. A circle with a cross inscribed in it indicates a vector pointing into and behind the diagram. These can be thought of as viewing the tip of an arrow head on and viewing the flights of an arrow from the back.

Vectors in n-dimensions

In order to calculate with vectors, the graphical representation may be too cumbersome. Vectors in an n-dimensional Euclidean space can be represented as

coordinate vectors in a cartesian coordinate system. The endpoint of a vector can be identified with an ordered list of n real numbers. These numbers are the coordinates of the endpoint of the vector, with respect to a given cartesian coordinate system, and are typically called as the scalar components of the vector on the axes of the coordinate system.



Applications:

Analytical geometry of three dimension tends to have very many different real life applications. One such application is in the field of chemistry where it is applied in order to help scientist understand the exact structure of the given crystal and a good example is the isometric crystals which are usually shaped as cubes

In geography, the concept of analytical geometry of three dimensions is often used to graph equations that usually model surfaces that are in shape for example the earth surface which is spherical.

In physics to determine tensions between different forces. Vectors can be used to find the angle between two adjacent sides of a grain elevator chute.

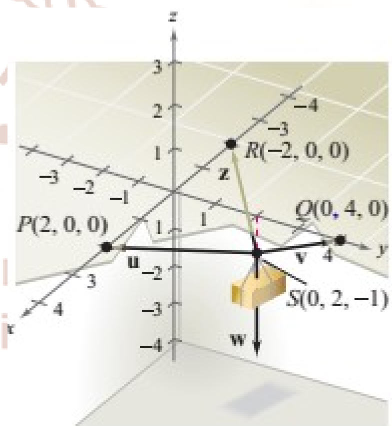
The following example problems show how to use vectors to solve an equilibrium problem in space.

Real life problems

1. represent tensions in cables to support auditorium lights

A weight of 480 pounds is supported by 3 ropes. The weight is located at S(0,2,-1). The ropes are tied to the points P(2,0,0), Q(0,4,0),and R(-2,0,0). Find the force (or tension) on each rope.

Solution:



The (Downward) force of the weight is represented by the vector.

w=<0,0,-480>.

The force vectors corresponding to the ropes are as follows:

$$\mathbf{u} = \|\mathbf{u}\| \frac{\overrightarrow{SP}}{\|\overrightarrow{SP}\|} = \|\mathbf{u}\| \frac{\langle 2-0, 0-2, 0-(-1) \rangle}{3}$$

$$= \|\mathbf{u}\| \left\langle \frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right\rangle$$

$$\mathbf{v} = \|\mathbf{v}\| \frac{\overrightarrow{SQ}}{\|\overrightarrow{SQ}\|} = \|\mathbf{v}\| \frac{\langle 0-0, 4-2, 0-(-1) \rangle}{\sqrt{5}}$$

$$= \|\mathbf{v}\| \left\langle 0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle$$

$$\mathbf{z} = ||\mathbf{z}|| \frac{\overrightarrow{SQ}}{||\overrightarrow{SQ}||} = ||\mathbf{z}|| \frac{<-2-0, 0-2, 0-(-1)>}{3} = ||\mathbf{z}|| \left\langle -\frac{2}{3}, -\frac{2}{3}, \frac{1}{3} \right\rangle$$

For the system to be in equilibrium, it must be true that

u + v + z + w = 0 or u + v + z = -w

This yields the following system of linear equations

$$\frac{2}{3}||\mathbf{u}|| + \quad -\frac{2}{3}||z|| = 0$$

$$-\frac{2}{3}||\mathbf{u}|| + \frac{2}{\sqrt{5}}||\mathbf{v}|| - \frac{2}{3}||\mathbf{z}|| = 0$$

$$\frac{1}{3}||\mathbf{u}|| + \frac{1}{\sqrt{5}}||\mathbf{v}|| + \frac{1}{3}||\mathbf{z}|| = 480$$

We can find the solution of the system to be

||u|| = 360.0

||v|| ≈ 536,.7

||z|| = 360.0

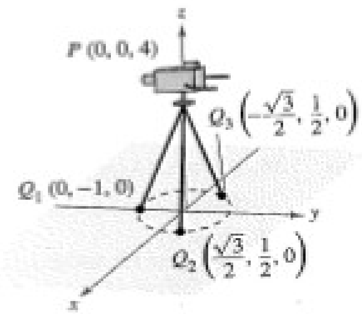
So the rope attached at point P has 360 pounds of tension, The rope attached at point Q has about 536.7 pounds of tension and the rope attached at point R has 360 pounds of tension.

2. Measure the force vector

A television camera weighting 120 pounds is supported by a tripod, Represent the force exerted on each leg of the tripod as a vector.

Solution:

Let the vectors F_1,F_2 and F_3 represent the forces exerted on the three legs. You can determine the directions of F_1, F_2 and F_3 as follows.



$$\overrightarrow{PQ_1} = \langle 0-0. -1-0, 0-4 \rangle$$

$$= \langle 0, -1, -4 \rangle$$

$$\overrightarrow{PQ_2} = \langle \frac{\sqrt{3}}{2} - 0, \frac{1}{2} - 0, 0-4 \rangle$$

$$= \langle \frac{\sqrt{3}}{2}, \frac{1}{2}. -4 \rangle$$

$$\overrightarrow{PQ_3} = \langle -\frac{\sqrt{3}}{2} - 0, \frac{1}{2} - 0, 0-4 \rangle$$

$$= \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}. -4 \rangle$$

Because each leg has the same length, and the total force is distributed equally among the three legs, you know that $||F_1|| = ||F_2|| = ||F_3||$. So there exist a constant c such that

$F_1 = c\langle 0, -1, -4 \rangle$

$F_2 = c\langle \frac{\sqrt{3}}{2}, \frac{1}{2}. -4 \rangle$

$F_3 = c\langle -\frac{\sqrt{3}}{2}, \frac{1}{2}. -4 \rangle$

Let the total Force exerted by object be given by **F** = $\langle 0, 0, -120 \rangle$. Then using the fact that

F= F_1+F_2 +F_3

You can conclude that F_1,F_2 and F_3 all have a vertical component of -40. This implies that c(-4)= 40 and c=10. Therefore, the force exerted on the legs can be represented by

$F_1 = \langle 0, -10, -40 \rangle$

$F_2 = c \langle 5\sqrt{3}, 5. -40 \rangle$

$F_3 = c \langle -5\sqrt{3}, 5. -40 \rangle$

3. Magnitude of the resultant force

A crane lifts a steel beam upward with the force of 5000N. At the same time, two workers push the beam with forces of 600N towards the west and 650 N towards the north. Determine the magnitude of the resultant force acting on the beam.

Solution:

The three forces are perpendicular to each other. Define the vertical vector as [0,0,5000].

The other two vectors are horizontal. Define the force to the west as [-600,0,0] and the force to the north as [0,650,0]

$\vec{R}$ = [0,0,5000] + [-600,0,0] + [0,650,0]

= [-600 , 650 , 5000]

$|R| = \sqrt{-600^2 + 650^2 + 5000^2}$

= 5077.6 N

The magnitude of the resultant force is 5078 N.

4. Plane's velocity

Vectors in space can be used to represent velocity in three dimension for example it can be used to represent the plane's velocity

A jet airplane just after takeoff is pointed due east. Its air velocity vector makes an angle of 30° with flat ground with an air speed of 250 mph. If the wind is out of the south east at 32mph, calculate a vector that represents the plane's velocity relatives to the point of takeoff.

Solution:

Let i points east, j point north, k point up. The plane's air velocity is

$a = \langle 250\cos 30°, 0, 250\, sin30° \rangle \approx \langle 216.506, 0, 125 \rangle$

And the wind velocity, which is pointing northwest, is

$w = n \langle 32\, cos135°, 32\, sin135°, 0 \rangle$

$\approx \langle -22.627, 22.627, 0 \rangle$

The velocity relative to the ground is **v=a+w,** so

$v \approx \langle 216.506, 0, 125 \rangle + \langle -22.627, 22.627, 0 \rangle$

$\approx \langle 193.88, 22.63, 125 \rangle$

= 193.88i +22.63j + 125k

Conclusion

The present study aimed to understand the concept of vectors in three dimensional analytic geometry. These study helps to find the physical tension, force and velocity through examples. The Result of the study shows that the importance of vectors in real life applications.

References:

1. www.math.tamu.edu>Math251
2. www.manualmath.info>downloads
3. https://www.cs.bham.acuk>pub>mas

4. Lutz, P. L. (992). The Age of Plato and Aristotle. Rise of Experimental Biology, the, (2), 25-34.
5. ManualMaths. (2014). Chapter 11 Analytic Geometry in Three Course Number Dimensions.
6. Mark, H., & Workman, J. (2007). Analytic Geometry: Part 1 – The Basics in Two and Three Dimensions. Chemo metrics in Spectroscopy, (4), 71-76.
7. Pekoe, D. (1988). Geometry, a comprehensive course. New York: Dover Publications
8. Potter, M. H., & Moray, C. B. (1985). Analytic Geometry in Three Dimensions. Intermediate Calculus Undergraduate Texts inmysite.science.uottawa.ca>iabde083