International Journal of Trend in Scientific Research and Development (IJTSRD)
Volume 10 Issue 1, Jan-Feb 2026 Available Online: www.ijtsrd.com e-ISSN: 2456 – 6470

Density Topology in Tritopological Spaces

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ABSTRACT

In this paper we introduce the concept of density topology in a tritopological space and derive some relevant separation properties involving the density topology.

KEYWORDS: *Tritopological spaces, Triowise Borel sets, Triowise closure, Density topology, Density of sets.*

Mathematics Subject Classification: 54D10, 54D15, 54A05, 54C08.

How to cite this paper: S. K. Biswas | N. Akhter "Density Topology in Tritopological Spaces" Published in International Journal of Trend in Scientific Research and Development (ijtsrd), ISSN: 2456-6470, Volume-10 | Issue-1, February 2026, pp.418-423, URL: www.ijtsrd.com/papers/ijtsrd100083.pdf

IJTSRD100083

1. INTRODUCTION

This is the forth in a series of our M. Phil papers. The first, the second and the third such papers have appeared in 2015[16], in 2016[17] and in 2024[18]. The idea of density topology has been widely studied in various spaces such as bitopological spaces, measure spaces, real number spaces, Romanvoski spaces etc. (see Goffman and Waterman (1961), Lahiri and Das (2002), Martin (1964), Saha and Lahiri (1989), A.K. Banerjee (2008). The concept of tritopological spaces are introduced in [8]. We have generalized a work of Lahiri and Das [2002] to tritopological spaces.

In this paper we attempt to define density of sets in a tritopological space (X, P, Q, R) and with the idea of trioclosure we generate a topology which is helpful in study of some separation properties. Here we study density of sets in a tritopologcial space satisfying certain axioms and investigate some relevant separation properties.

We have used the definitions and terminology of text book of S. Majumdar and N. Akhter [1], Munkres [2], Dugundji [3], Simmons [4], Kelley [5] and Hocking-Young [6].

2. Tritopological Spaces

Definition 2.1: Let X be a non-empty set. If P, Q, R are three collections of subsets of X such that (X, P), (X, Q) and (X, R) are three topological spaces then X will be called a tritopological space and will be denoted by (X, P, Q, R).

Example 2.1: Let X = $\{a, b, c, d\}$

P $= \{X, \phi, \{c, d\}\}$, Q $= \{X, \phi, \{a, b, c\}, \{b\}\}$

and R $= \{X, \phi, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}\}$

then (X, P), (X, Q) and (X, R) are three

topological spaces and (X, P, Q, R) is a tritoplogical space.

Here we give some definitions in tritopological spaces.

Definition 2.2: A cover Ω of a tritoplogical space (X, P, Q, R) is said to be trio-pen if $\Omega \subset P \cup Q \cup R$ and Ω contains at least one non-empty member from each of P, Q, and R . The space (X, P, Q, R) is said to be tricompact if every trio-open cover of it has a finite subcover.

Definition 2.3: Let (X, P, Q, R) be a tritopological space. For any $A \subset X$, define $\overline{A} = \cap \{F_1 \cup F_2 \cup F_3$ where $A \subset F_1 \cup F_2 \cup F_3$ and F_1, F_2 and F_3 are respectively P, Q and R- closed$\}$, then $\overline{A}$ is called the triowise closure of A.

N.B: When A is a subset of a tritopological space X (bitopological space X) by $\overline{A}$ we mean triowise closure of A (pairwise closure of A).

Theorem 2.1: Let X be a tritopological space and let $S=\{V: V \subset X$ and $\overline{(X-V)} = X - V\}$, then (X, S) is a topological space.

Note: We observe that if F is P or Q or R closed then $\overline{F} \subset F \cup \varphi \cup \varphi = F$, so that $\overline{F} = F$. Hence F is S–closed. This implies that S is finer than P, Q and R.

Lemma 2.1: The family $\{P \cap Q \cap R: P \in P, Q \in Q$ and $R \in R\}$ forms a base for S.

Proof: Clearly, the sets $P \cap Q \cap R$ belong to S. If $V \in S$ then

$X - V = \overline{(X - V)}$

$= \cap \{ F_1 \cup F_2 \cup F_3: F_1 \cup F_2 \cup F_3 \supset X - V$ and F_1, F_2 and F_3 are respectively P, Q and R- closed$\}$

Therefore, $V = X - (X - V)$

$= \cup \{P \cap Q \cap R: P \cap Q \cap R \subset V, P \in P, Q \in Q$ and $R \in R\}$

Hence the proof is complete.

3. Density of sets in (X, P, Q, R)

Definition 3.1: $B \subset P \cup Q \cup R$ is said to be triowise open base of (X, P, Q, R) if $B \cap P$ form a base for P, $B \cap Q$ form a base for Q and $B \cap R$ form a base for R.

Definition 3.2: The σ-algebra generated by the class of all sets of the form $P \cup Q \cup R$, $P \in P$, $Q \in Q$ and $R \in R$ is called the class of triowise Borel sets.

Definition 3.3: A mapping f : (X, P, Q, R) → (Y, P_1, Q_1, R_1) is said to be triowise continuous if inverse image of every P_1-open (resp. Q_1-open, R_1-open) set in Y is P-open (resp. Q-open, R-open) in X.

In (X, P, Q, R), let ξ be the class of all triowise Borel sets. Let μ be a measure on ξ such that μ (X) is finite. We also assume μ to be non-zero for all non-void sets of the form $P \cap Q \cap R$, $P \in P$, $Q \in Q$, $R \in R$. Let μ* be the outer measure on P(X) generated by μ. Let ϑ be the class of all μ*-measurable sets and A be a class of sets from ξ.

Definition 3.4: By a decomposition [14] ξ_V of $V \in \xi$ we mean a finite disjoint family

$\{A_1, A_2, \ldots, A_n\} \subset A$ such that

(i) $\bigcup_{i=1}^{n} Ai \subset V$ *and*

(ii) $\mu(V - \bigcup_{i=1}^{n} Ai) = 0$

The class A is called a triowise fundamental sets [14] if the following axioms hold.

AXIOM I. A form a triowise open base of (X, P, Q, R) (and hence also $A \subset P \cup Q \cup R$).

AXIOM II. For any $A \in A$ and $\in > 0$ there is a decomposition ξ_A of A such that $A' \in \xi_A$ implies $\mu(A') < \in$.

AXIOM III. For each triowise compact set W and for each P or, Q or R-open set $V \supset W$, there is an $\in > 0$ such that if $A \in A$ and $\mu(A) < \in$ and $\overline{A} \cap W \neq \phi$ then $A \subset V$.

AXIOM IV: Given $A \in A$ and $\in > 0$ there is a $A' \in A$ such that $\overline{A} \subset A'$ and $\mu(A' - A) < \in$.

Let $x \in X$, since by Axiom I A forms a triowise open base of (X, P, Q, R), there exists a $A \in A$ such that $x \in A$. Let $\in > 0$, by AXIOM II there exists a decomposition $\{A_1, A_2, \ldots, A_n\}$ of A with $\mu(A_i) < \in$, i=1, 2, ..., n. We now prove that $x \in \overline{A_i}$ for some i. If not, let $x \notin \overline{A_i}$ for all i. Since $\overline{A_i}$ is the intersection of all S-closed sets containing A_i, there exists a S-closed set $\hat{A}_i$, containing A_i which does not contain x. Then clearly the set $G = A - \bigcup_{i=1}^{n} \hat{A}_i$ is S-open and non-void ($x \in G$) and $G \subset A - \bigcup_{i=1}^{n} Ai$ and so by our assumption about μ, $0 < \mu(G) \leq \mu(A - \bigcup_{i=1}^{n} Ai)$ which contradicts the condition (ii) of definition 3.4.

Hence for $\in > 0$ there exists a A_i such that $x \in \overline{A_i}$ and $\mu(A_i) < \in$.

Consequently, for each $x\in X$ there exists a sequence of triowise fundamental sets $\{A_{n,x}\}$ such that $x\in \overline{A}_{n,x}$ and $\mu(A_{n,x})<\frac{1}{n}$ $\forall$ n.

Definition 3.5: [15] For $x\in X$ and $E\subset X$ the upper and lower outer density of E at x denoted respectively by $\overline{\varphi}^*(E, x)$, $\underline{\varphi}^*(E, x)$ are defined by

$$\overline{\varphi}^*(E,x)=\lim_{n\to\infty}\overline{\varphi}^*_n(E,x)$$

$$\underline{\varphi}^*(E,x)=\lim_{n\to\infty}\underline{\varphi}^*_n(E,x)$$

where,

$\overline{\varphi_n}^*(E,x)=$ sup $\{m^*(E, A);\ x\in \overline{A},\ \mu(A)<\frac{1}{n},$ $A\in \mathcal{A}\}$

$\underline{\varphi_n}^*(E,x)=$ inf $\{m^*(E, A);\ x\in \overline{A},\ \mu(A)<\frac{1}{n},$ $A\in \mathcal{A}\}$

and $m^*(E, A) = \dfrac{\mu^*(E\cap A)}{\mu(A)}$.

Clearly $0\le\underline{\varphi}^*(E,x)\le\overline{\varphi}^*(E,x)\le 1$. If they are equal, we denote the common value by $\varphi^*(E, x)$ and say the outer density of E exists at x. If $E\in\vartheta$ we write $\overline{\varphi}^*(E,x)=\overline{\varphi}(E,x)$ and $\underline{\varphi}^*(E,x)=\underline{\varphi}(E,x)$. If they are equal we write $\overline{\varphi}(E,x)=\underline{\varphi}(E,x)=\varphi(E,x)$.

We say x, an outer density point or an outer dispersion point of E according as $\underline{\varphi}^*(E, x) = 1$ or $\overline{\varphi}^*(E,x)=0$.

Theorem 3.1:[Theorem 4.1 of [13]] If $E, F\in\vartheta$, $\varphi(E,x)$, $\varphi(F,x)$ exist and if $E\subset F$, then $\varphi(F-E,x)$ exists and $\varphi(F-E,x) = \varphi(F,x)-\varphi(E,x)$.

The proof is similar to the proof of theorem 3 of Saha and Lahiri (1989).

Definition 3.6: Let $D= \{V:V\subset X$ and $\varphi^*(X-V,x)=0,\ \forall x\in V\}$. As in Martin (1964) one can verify that D is a topology on X which is called the density topology (or, in short d-topology) on (X, P, Q, R).

The following two theorems is a generalizations of theorems 3 and 4 of [15].

Theorem 3.2: If V is S-open, then $\forall$ $x\in V$ the outer density of V exist at x and $\varphi^*(V,x)=1$.

Proof: Let $x\in V$, so by Lemma 2.5 there exists $P\in \mathcal{P}$, $Q\in \mathcal{Q}$, $R\in \mathcal{R}$ such that $x\in P\cap Q\cap R\subset V$. Since $\{x\}$ triowise compact and $\{x\}\subset P$, by Axiom III, there is $\epsilon>0$ such that $A\in \mathcal{A}$ and $\{x\}\cap\overline{A}\neq\phi$ and $\mu(A)<\epsilon \Rightarrow A\subset P$. Choose $n_0\in N$ such that $1/n_0<\epsilon$. Then $\forall$ $n>n_0$, $A\in\mathcal{A}$ and $x\in\overline{A}$ and $\mu(A)<1/n$ will imply $A\subset P$. Similarly, we can find $n_1\in N$ such that $\forall$ $n\ge n_1$, $A\in\mathcal{A}$ and $x\in\overline{A}$ and $\mu(A)<1/n\Rightarrow A\subset Q$ and $n_2\in N$ such that $\forall$ $n>n_2$, $A\in\mathcal{A}$ and $x\in\overline{A}$ and $\mu(A)<1/n\Rightarrow A\subset R$. Then $\forall$ $n\ge m = \max\{n_0, n_1, n_2\}$, $A\in\mathcal{A}$ and $x\in\overline{A}$ and $\mu(A)<1/n\Rightarrow A\subset P\cap Q\cap R\subset V$. Hence from definition of $\underline{\varphi_n}^*(V, x)$ it follows that $\forall$ $n\ge m$.

$$\underline{\varphi_n}^*(V,x)=\inf \frac{\mu^*(V\cap A)}{\mu(A)}, A\in\mathcal{A},\ x\in\overline{A}, \mu(A)<1/n$$

$= 1$, since $A\subset V$.

Therefore $\underline{\varphi}^*(V,x)=1$ and hence

$\varphi^*(V,x)=1$.

Theorem 3.3: The d- topology D is finer than S.

Proof: Since $S\subset\xi\subset\vartheta$ so by theorems 3.1 and 3.2 $V\in S$ implies

$$\varphi(X-V,x)=\varphi(X,x)-\varphi(V,x)$$
$$=1-1$$
$$=0\ \forall\ x\in V.$$

Therefore $S\subset D$. This completes the proof.

4. Separation Properties in (X, S)

Theorem 4.1: [Theorem 5 of [15]] (X, S) is regular.

Proof: Let E be S-closed and $x\notin E$. Then $x\in X-E\in S$ and so by Lemma 2.1, there are $P\in\mathcal{P}$, $Q\in\mathcal{Q}$ and $R\in\mathcal{R}$ such that $x\in P\cap Q\cap R\subset X-E$. We associate with x, a sequence of trio wise fundamental sets $\{A_{n},x\}$ such that $x\in\overline{A}_{n,x}$ and $\mu(A_{n,x})<1/n$ $\forall$ n. By Axiom IV for $A_{2n,x}$ there is $B_{n,x}\in$ A and a S-closed sets $\hat{A}_{2n,x}$ such that $x\in\overline{A}_{2n,x}\subseteq A_{2n,x}\subset B_{n,x}$ and $\mu(B_{n,x}-A_{2n,x})<1/2n$. Then $\mu(B_{n,x})\le\mu(A_{2n,x})+\mu(B_{n,x}-A_{2n,x}) <\frac{1}{2n}+\frac{1}{2n}=1/n$. Thus we obtain a sequence $\{B_{n,x}\}$ from A such that $x\in B_{n,x}$ and $\mu(B_{n},x)<1/n$ $\forall$n. Again, proceeding as above for $B_{2n,x}$ we get a sequence $\{C_{n,x})$ from A satisfying $x\in B_{2n,x}\subset\overline{B}_{2n,x}\subset C_{n,x}$ and $\mu(C_{n,x})<1/n$ $\forall$ n. Since $\{x\}$ is triwise compact and $x\in p$ so by Axiom III, there is $\in >0$ such that $A\in$ A and $\{x\}\cap\overline{A} \neq \varphi$ and μ (A) $<\in$ implies $A \subset P$. Choose $n_o \in \aleph$ such that $1/n_\circ < \in$. Then $C_{no,}\ x \subset P$, since $x\in C_{n0,\ x} \subset \overline{C}_{n^\circ}$, x and μ $(C_{n0}, x) < 1/_{n0} < \in$. Now, $x\in B_{2n0}, x \subset \hat{B}_{2n0,}\ x \subset C_{n0}, x \subset P$.

Similarly, we can find $n_1\in \aleph$ such that $x\in B_2n$, $x \subset B_{2n}, x \subset C_{n1}$, $x \subset Q$ and for $n_2\in\aleph$ such that $x\in B_2n_2$, $x \subset \hat{B}_{2n2}$, $x\subset C_{n2}, x \subset R$.

Thus $x\in B_{2n0},x \cap B_{2n1},x,\cap B_{2n2},x = U$ (say) $\subset \hat{B}_{2n0,}x \cap \hat{B}_{2n1},x \cap \hat{B}_{2n2},x = F$ (say) $\subset Cno,x \cap C_{n1},x \cap C_{n2},x \subset p\cap Q\cap R \subset X\text{-}E$, Hence we have $U\in S$ (by Axiom I), $V = X\text{-}F \in S$ satisfying $x\in U$, $E\subset V$ and $U\cap V = \phi$. Hence proved.

Corollary 4.1: [13] If (X, S) is T_0 then it is T_2.

Proof: Let, x, y be two distinct points of X, then by T_0 there is a S-open set U containing one of them say x, such that $y\in F = X\text{-}U$. Since F is S-closed and $x\notin F$, so by regularity there are S-open sets V,W such that $x\in V$, $F\subset W$ and $V\cap W=\phi$. Then $x\in V$, $y\in W$ and $V\cap W= \phi$. Therefore (X, S) is T_2.

Definition 4.1: In (X, P, Q, R), P is said to be regular w.r.to Q if for any P- closed set F and $x \in X$ with $x\notin F$, there exist $U \in P$, $V\in Q$ such that $x\in U$, $F\subset V$ and $U\cap V = \phi$. If P is regular w.r.to Q, and Q is regular w.r.to R and R is regular w.r.to P then (X, P, Q, R) is called (1,2,3)-regular. If the space (X, P, Q, R) is (i, j, k)- regular then it is called triowise regular for $i \neq j \neq k$, i, j, k = 1,2,3.

If we consider the bitopological space the (X, P, Q) and define $T = \{U: U\subset X$ and $\overline{X-U} = X\text{-}U\}$ then (X,T) is a topological space by Lahiri and Das [2002].

This topology T is finer than both P and Q. By Lemma 2 of Lahiri and Das [2002], we have $\{P \cap Q$: $P \in$ P and $Q\in$ Q$\}$ form a base for T.

Therefore, by Lemma 2.1, S is finer than T.

Also (X, T) is regular by Lahiri and Das [2002].

Then we have the following corollary.

Corollary 4.2: In (X, D, S, T) S is regular with respect to D and T is regular with respect to S and D.

Note: When we say that T is regular with respect to D, without loss of generality, we assume that D is the d- topology on (X, P, Q).

Theorem 4.2: (X, D, S, T) is triowise regular if the following condition:

(a) For any D-closed set E, if $\{A_n\}$ is a sequence of triowise fundamental sets such that $\mu^*(E\cap A_n)\to 0$ as $n\to\infty$, there is at least one $k\in N$ such that $E\cap A_k=\varphi$ holds.

Proof: We only need to show that in X, T is regular with respect to S, S is regular with respect to D and D is regular with respect to T. By corollary 4.4 it is sufficient to show that D is regular with respect to T. Let E be D-closed and $x\notin E$. As in Theorem 4.1 we construct two sequences $\{B_{n,x}\}$, $\{C_{n,x}\}$ from A such that $x\in B_{2n,x} \subset \overline{B}_{2n,x} \subset C_{n,x}$ and μ $(C_{n,x}) < {}^1/_n$ $\forall$ n.

Since X-E is D-open and $x\in X\text{-}E$, $\overline{\varphi}^*(E,x)=0$. Let $\in >0$ be arbitrary. Then there is $n_0 \in N$ such that ${}^1/_n < \in$ and $\overline{\varphi_n}^*(E,x) < \in \forall\ n\geq n_0$. Since $x\in C_{n,x} \subset \overline{C}_{n,x}$, μ $(C_{n,x}) < {}^1/_n$, m^* $(E, C_{n,x}) < \in \forall n\geq n_0$ i.e, $\mu^*(E\cap C_{n,x})/\mu(C_{n,x}) < \in$ i.e; $\mu^*(E\cap Cn,x) < \in.\ \mu(C_n,x) < \in\ \forall\ n\geq_{n0}$. Thus $\mu^*(E\cap C_{n,x})\to 0$ as $n\to\infty$. By the condition (a) there is $k\in N$ such that $E\cap C_{k,x}= \phi$. Hence using Axiom I, $x\in B_{2k,x}\in T\subset D$, $E\subset X\text{-}C_{k,x}\subset X\text{-}\overline{B}_{2k,x}\in T$ and $B_{2k},x \cap (X\text{-}\overline{B}_{2k,x})= \phi$. This proves the theorem.

Definition 4.2: (X, P, Q, R) is said to be triowise Hausdorff if for every x, y, $z \in X$, $x\neq y\neq z$ there exist $U \in P$, $V\in Q$ and $W\in R$ such that $x\in U$, $y\in V$, $z\in W$ and $U\cap V\cap W= \phi$.

Example 4.1: Consider the following tritopologies on $X = \{a,b,c\}$:

$P_1 = \{X,\phi,\{a\},\{a,b\}\}$

$P_2 = \{X,\phi,\{b,c\},\{b\}\}$

$P_3 = \{X,\phi,\{a,c\},\{c\}\}$

Then X is (1,2,3) Hausdorff since for $a,b,c\in X$, P_1- open set U = $\{a\}$, P_2- open set V = $\{b\}$, P_3- open set W = $\{c\}$, then we have $U\cap V\cap W=\phi$.

Theorem 4.3: If (X, P) or (X, Q) or (X, R) is T_1, then (X, D, T) is pairwise Hausdorff. Also if (X, D, T) is pairwise Hausdorff then (X, D, S, T) is triowise Hausdorff.

Proof: The proof of the first part is similar to Theorem 7 of Lahiri and Das [2002].

For the last part of the theorem, let (X, D, T) be pairwise Hausdorff. Let x, y, $z\in X$ with $x\neq y\neq z$. Since (X, D, T) is pairwise Hausdorff, for x, y there exist $U'\in D$, $V'\in T$ such that $x\in U'$, $y\in V'$ and $U'\cap V'= \phi$, for y, z there exist $U''\in D$, $V''\in T$ such that $y\in U''$, $z\in V''$ and $U''\cap V''= \phi$ and also for x, z there exist

$U''' \in D$, $V''' \in T$ such that $x \in U'''$, $z \in V'''$ and $U''' \cap V''' = \phi$. Put $U' \cap U''' = U$, $V' = V$ and $V'' \cap V''' = W$. Then $x \in U \in D$, $y \in V \in T \subset S$, $z \in W \subset T$ and clearly, $U \cap V \cap W = \phi$.

Therefore (X, D, S, T) is triowise Hausdorff.

Definition 4.3: (X, P, Q, R) is called (1,2,3) normal if for any pairwise disjoint P-closed set A, Q-closed set B, R-closed set C, there exist $U \in P$, $V \in Q$, $W \in R$ such that $A \subset V$, $B \subset W$, $C \subset U$ and $U \cap V \cap W = \phi$.

If (X, P, Q, R) is (i, j, k) normal then it is called triowise normal for $i \neq j \neq k$, i,j,k=1,2,3.

Theorem 4.4: If (X, D) is compact then (X, D, T, S) is triowise normal.

Proof: Let A, B and C be pairwise disjoint D-closed, T -closed and S-closed sets respectively. Since (X, S) is regular for any $x \in A$, there exist U_x, $V_x \in S$ such that $x \in U_x$, $C \subset V_x$ and $U_x \cap V_x = \phi$. Now, $\{U_x : x \in A\}$ form a S-open cover of A and hence D-open cover of A.

Since A is D-closed there exist $x_1, x_2, \ldots, x_n \in A$ such that $A \subset \bigcup_{i=1}^{n} U x_i = U' \in S$, $C \subset \bigcap_{i=1}^{n} V x_i = W \in S \subset D$, $U' \cap V = \phi$. Also since (X, T) is regular for each $x \in A$, there exist $U'x$, $V'x \in T$ such that $x \in U'x$, $B \subset V'x$ and $U'x \cap V'x = \phi$. Also $\{U'x : x \in A\}$ form a T open cover of A and hence D-open cover of A. Since A is D-closed $A \subset \bigcup_{i=1}^{n} U' x_i = U \in T$, $B \subset \bigcap_{i=1}^{n} V' x_i = V \in U' \in T \subset S$, $U \cap W = \phi$. Therefore, we have $A \subset U \in T$, $B \subset V \in S$, $C \subset W \in D$ and $U \cap V \cap W = \phi$.

Thus (X, D, T, S) is (1,2,3) normal. Similarly, we can show that (X, D, T, S) is (2,1,3) normal. Therefore (X, D, T, S) is triowise normal.

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