International Journal of Trend in Scientific Research and Development (IJTSRD)
UGC Approved International Open Access Journal
ISSN No: 2456 - 6470 | www.ijtsrd.com | Volume - 1 | Issue – 5

Comparison of Septic and Octic Recursive B-spline Collocation Solutions for Seventh Order Differential Equation

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ABSTRACT

This note is concerned of improvement in numerical solution for seventh order linear differential equation by using the higher degree B-spline collocation solution than its order. Numerical examples are used to study the improvement in the accuracy of the numerical solutions with the high degree B-spline base function.

Keywords: *B-Spline, Collocation, Recursive, Linear differential equation*

1. INTRODUCTION

In this note we consider Septic and Octic B-spline collocation methods for seventh order linear boundary value problem.

$$P_1(x)\frac{d^7U}{dx^7}+P2(x)\frac{d^6U}{dx^6}+P3(x)\frac{d^5U}{dx^5}+P_4(x)\frac{d^4U}{dx^4}+P5(x)\frac{d^3U}{dx^3}+P_6(x)\frac{d^2U}{dx^2}+P_7(x)\frac{dU}{dx}+P_8(x)U=Q(x)$$

$x\in(a,b)$, (1)

with the boundary conditions

i) $U(a)=d1, U(b)=d2 \;\; U'(a)=d3, U'(b)=d4, U''(a)=d5, U''(b)=d6, U'''(a)=d7$

where $a,b,d_1,d_2,d_3,d_4,d_5,d_6,d_7$ **are constants.** $P_1(x), P_2(x), P_3(x), P_4(x), P_5(x), P_6(x)$, $P_7(x), P_8(x), Q(x)$ **are function of** x**.**

2. THE NUMERICAL SCHEME

Let $[a,b]$ be the domain of the governing differential equation and is partitioned as $X=\{a=x_0,x_1,x_2 \ldots\ldots\ldots x_{n-1},x_n=b\}$ without any restriction on length of n sub domains. Let $N_i(x)$ be septic B- splines with the knots at the points x_i, i=0,1,…n. The set $\{N_{-7}, N_{-6}, N_{-5}, \ldots, N_6,$

N_7 } forms a basis for functions defined over [a,b].Two more points should be included both sides for knot set to evaluate Octic B-spline over [a,b] to maintain the partition of unity property of B-splines.

Let $U^h(x) = \sum_{i=-7}^{n+7} C_i N_{i,p}(x)$ -- (2)

where C_i's are constants to be determined and $N_{i,p}(x)$ are the septic B-spline functions , be the approximate global solution to the exact solution $U(x)$ of the considered seventh order linear differential equation (1).

A zero degree and other than zero degree B-spline basis functions are defined at x_i recursively over the knot vector space $X = \{x_1, x_2, x_3 .... x_{n-1}, x_n\}$ as

i) if $p = 0$

$$N_{i,p}(x) = 1 \quad \text{if} \quad x \in (x_i, x_{i+i})$$
$$N_{i,p}(x) = 0 \quad \text{if} \quad x \notin (x_i, x_{i+i})$$

ii) if $p \geq 1$

$$N_{i,p}(x) = \frac{x - x_i}{x_{i+p} - x_i} N_{i,p-1}(x) + \frac{x_{i+p+1} - x}{x_{i+p+1} - x_{i+1}} N_{i+1,p-1}(x) \quad .... \quad (3)$$

where p is the degree of the B-spline basis function and x is the parameter belongs to X .When evaluating these functions, ratios of the form 0/0 are defined as zero

Derivatives of B-splines

If p=7, we have

$$N'_{i,p}(x) = \frac{x - x_i}{x_{i+p} - x_i} N'_{i,p-1}(x) + \frac{N_{i,p-1}(x)}{x_{i+p} - x_i} + \frac{x_{i+p+1} - x}{x_{i+p+1} - x_{i+1}} N'_{i+1,p-1}(x) - \frac{N_{i+1,p-1}(x)}{x_{i+p+1} - x_{i+1}}$$

$$N^{vii}_{i,p}(x) = 7\frac{N^{vi}_{i,p-1}(x)}{x_{i+p} - x_i} - 7\frac{N^{vi}_{i+1,p-1}(x)}{x_{i+p+1} - x_{i+1}} \quad .... \quad (4)$$

$$(U^h)^{vii}(x) = \sum_{i=-7}^{n+7} C_i N^{vii}_{i,p}(x) \quad .... \quad (5)$$

The x_i's are known as nodes, the nodes are treated as knots in collocation B-spline method where the B-spline basis functions are defined and these nodes are used to make the residue equal to zero to determine unknowns C_i's in (2).Seven extra knots are taken into consideration besides the domain of problem when evaluating the septic B-spline basis functions at the nodes which are within the considered domain.

Substituting the equations (2) and (5) in equation (1) for U (x) and derivatives of U (x).Then system of (n+1) linear equations are obtained in (n+8) constants. Applying the boundary conditions to equation (2) , seven more equations are generated in constants. Finally, we have (n+8) equations in (n+8) constants.

Solving the system of equations for constants and substituting these constants in equation (2) then assumed solution becomes the known approximation solution for equation (1) at corresponding the collocation points.

This is implemented using the Matlab programming.

3. NUMERICAL EXAMPLES

$$\frac{d^7 y}{dx^7} + y = -e^x\,(35 + 12x + 2x^2)$$ with the boundary conditions

$$y(0)=0,\ y(1)=0, y'(0)=1,\ y'(1)=-e,\ y''(0)=0, y''(1)=-4e,$$

$$y'''(0)=-3$$

The exact solution is $y = x\,(1-x)\,e^x$

Table1: Comparison of Seventh degree B-spline collocation solution with the Exact solution

Nodes	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
SBCS*	0	0.0995	.1954	.2835	.3580	.4122	.4373	.4229	.3561	.2214	0
Exact Solution	0	0.0995	.1954	.2835	.3580	.4122	.4373	.4229	.3561	.2214	0

* Septic B-spline collocation solution

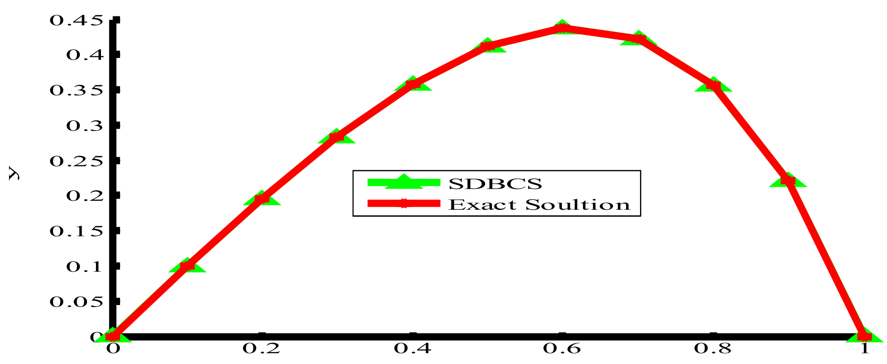

Figure1: Comparison of Septic collocation solution with the exact solution

Table2: Maximum relative errors of SBCS and OBCS for different number of nodes

Number of nodes	11	21	31
Max relative error SCBS	1.5849e-005	8.2272e-006	5.5212e-006
Max relative error OBCS	3.5520e-007	8.9814e-008	5.5680e-008

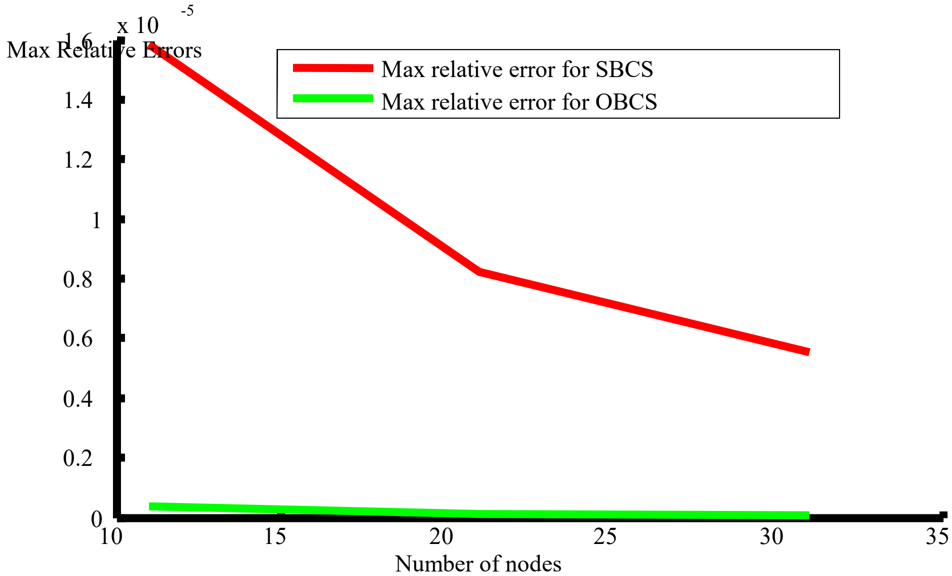

Figure2: Comparison of maximum relative errors for SBCS and OBCS

Conclusion

A Septic and Octic degree B-spline functions are used as basis in collocation method to find numerical solutions for seventh order linear differential equation with the boundary conditions problem. It is observed that the Octic B-spline collocation solution gives the more accurate solution than septic B-spline collocation solution.The accuracy achieved by octic B-spline collocation method is done septic B-spline collocation method by taking much number of collocation points.

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