Contra $\mu$-$\beta$-Generalized $\alpha$-Continuous Mappings in Generalized Topological Spaces

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ABSTRACT
In this paper, we have introduced contra $\mu$-$\beta$-generalized $\alpha$-continuous maps and also introduced almost contra $\mu$-$\beta$-generalized $\alpha$-continuous maps in generalized topological spaces by using $\mu$-$\beta$-generalized $\alpha$-closed sets (briefly $\mu$-$\beta$GaCS). Also we have introduced some of their basic properties.

Keywords: Generalized topology, generalized topological spaces, $\mu$-$\alpha$-closed sets, $\mu$-$\beta$-generalized $\alpha$-closed sets, $\mu$-$\alpha$-continuous, $\mu$-$\beta$-generalized $\alpha$-continuous, contra $\mu$-$\alpha$-continuous, almost contra $\mu$-$\beta$-generalized $\alpha$-continuous.

1. INTRODUCTION
In 1970, Levin [6] introduced the idea of continuous function. He also introduced the concepts of semi-open sets and semi-continuity [5] in a topological space. Mashhour [7] introduced and studied $\alpha$-continuous function in topological spaces. The notation of $\mu$-$\beta$-generalized $\alpha$-closed sets (briefly $\mu$-$\beta$GaCS) was defined and investigated by Kowsalya, M and Jayanthi. D[4]. Jayanthi, D [2, 3] also introduced contra continuity and almost contra continuity on generalized topological spaces. In this paper, we have introduced contra $\mu$-$\beta$-generalized $\alpha$-continuous maps.

2. PRELIMINARIES
Let us recall the following definitions which are used in sequel.

Definition 2.1: [1] Let $X$ be a nonempty set. A collection $\mu$ of subsets of $X$ is a generalized topology (or briefly GT) on $X$ if it satisfies the following:
1. $\emptyset, X \in \mu$ and
2. If $\{M_i : i \in I\} \subseteq \mu$, then $\bigcup_{i \in I} M_i \in \mu$.

If $\mu$ is a GT on $X$, then $(X, \mu)$ is called a generalized topological space (or briefly GTS) and the elements of $\mu$ are called $\mu$-open sets and their complement are called $\mu$-closed sets.

Definition 2.2: [1] Let $(X, \mu)$ be a GTS and let $A \subseteq X$. Then the $\mu$-closure of $A$, denoted by $c_\mu(A)$, is the intersection of all $\mu$-closed sets containing $A$.

Definition 2.3: [1] Let $(X, \mu)$ be a GTS and let $A \subseteq X$. Then the $\mu$-interior of $A$, denoted by $i_\mu(A)$, is the union of all $\mu$-open sets contained in $A$.

Definition 2.4: [1] Let $(X, \mu)$ and $(Y, \mu_1)$ be GTSs. Then a mapping $f: (X, \mu_1) \to (Y, \mu_2)$ is called
i. $\mu$-semi-closed set if $i_\mu(c_\mu(A)) \subseteq A$
ii. $\mu$-pre-closed set if $c_\mu(i_\mu(A)) \subseteq A$
iii. $\mu$-$\alpha$-closed set if $c_\mu(i_\mu(c_\mu(A))) \subseteq A$
iv. $\mu$-$\beta$-closed set if $i_\mu(c_\mu(i_\mu(A))) \subseteq A$
v. $\mu$-regular-closed set if $A = c_\mu(i_\mu(A))$

Definition 2.5: [7] Let $(X, \mu_1)$ and $(Y, \mu_2)$ be GTSs. Then a mapping $f: (X, \mu_1) \to (Y, \mu_2)$ is called
i. $\mu$-Continuous mapping if $f^{-1}(A)$ is $\mu$-closed in $(X, \mu_1)$ for each $\mu$-closed in $(Y, \mu_2)$.
ii. $\mu$-Semi-continuous mapping if $f^{-1}(A)$ is $\mu$-semi-closed in $(X, \mu_1)$ for every $\mu$-closed in $(Y, \mu_2)$.
iii. $\mu$-pre-continuous mapping if $f^{-1}(A)$ is $\mu$-pre-closed in $(X, \mu_1)$ for every $\mu$-closed in $(Y, \mu_2)$.
iv. $\mu$-$\alpha$-continuous mapping if $f^{-1}(A)$ is $\mu$-$\alpha$-closed in $(X, \mu_1)$ for every $\mu$-closed in $(Y, \mu_2)$.
v. $\mu$-$\beta$-continuous mapping if $f^{-1}(A)$ is $\mu$-$\beta$-closed in $(X, \mu_1)$ for every $\mu$-closed in $(Y, \mu_2)$.
Definition 2.6: [9] Let \((X, \mu_1)\) and \((Y, \mu_2)\) be GTSs. Then a mapping \(f: (X, \mu_1) \rightarrow (Y, \mu_2)\) is called

i. contra \(\mu\)-Continuous mapping if \(f^{-1}(A)\) is \(\mu\)-closed in \((X, \mu_1)\) for every \(\mu\)-open in \((Y, \mu_2)\).

ii. contra \(\mu\)-semi continuous mappings if \(f^{-1}(A)\) is \(\mu\)-semi closed in \((X, \mu_1)\) for every \(\mu\)-open in \((Y, \mu_2)\).

iii. contra \(\mu\)-pre-continuous mappings if \(f^{-1}(A)\) is \(\mu\)-pre closed in \((X, \mu_1)\) for every \(\mu\)-regular closed set \(A\) of \((Y, \mu_2)\).

iv. contra \(\mu\)-\(\alpha\)-continuous mapping if \(f(A)\) is \(\mu\)-\(\alpha\)-closed in \((X, \mu_1)\) for every \(\mu\)-open in \((Y, \mu_2)\).

v. contra \(\mu\)-\(\beta\)-continuous mapping if \(f^{-1}(A)\) is \(\mu\)-\(\beta\)-closed in \((X, \mu_1)\) for every \(\mu\)-open in \((Y, \mu_2)\).

Example 3.7: Let \(A = \{c\}\) be a \(\mu\)-open set in \((Y, \mu_2)\). Then \(f^{-1}(\{c\})\) is a \(\mu\)-\(\beta\)-generalized \(\alpha\)-closed set in \((X, \mu_1)\).

Hence \(f\) is a contra \(\mu\)-\(\beta\)-generalized \(\alpha\)-continuous mapping.

Theorem 3.3: Every contra \(\mu\)-continuous mapping is a contra \(\mu\)-\(\beta\)-generalized \(\alpha\)-continuous mapping but not conversely in general.

Proof: Let \(f: (X, \mu_1) \rightarrow (Y, \mu_2)\) be a contra \(\mu\)-continuous mapping. Let \(A\) be any \(\mu\)-open set in \((Y, \mu_2)\). Since \(f\) is a contra \(\mu\)-continuous mapping, \(f^{-1}(A)\) is a \(\mu\)-closed set in \((X, \mu_1)\). Since every \(\mu\)-closed set is a \(\mu\)-\(\beta\)-generalized \(\alpha\)-closed set, \(f^{-1}(A)\) is a \(\mu\)-\(\beta\)-generalized \(\alpha\)-closed set in \((X, \mu_1)\). Hence \(f\) is a contra \(\mu\)-\(\beta\)-generalized \(\alpha\)-continuous mapping.

Example 5.1.4: Let \(X = Y = \{a, b, c, d\}\) with \(\mu_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}\) and \(\mu_2 = \{\emptyset, \{d\}, Y\}\). Let \(f: (X, \mu_1) \rightarrow (Y, \mu_2)\) be a mapping defined by \(f(a) = a, f(b) = b, f(c) = c, f(d) = d\). Now, \(\mu\)-\(\beta\)O(X) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, c, d\}\}

\(\mu\)-\(\beta\)O(Y) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{a, c, d\}\}. Let \(A = \{d\}\) be a \(\mu\)-open set in \((Y, \mu_2)\). Then \(f^{-1}(\{d\})\) is a \(\mu\)-\(\beta\)-generalized \(\alpha\)-closed set, but not \(\mu\)-closed as \(c_\beta(f^{-1}(A)) = c_\beta(\{d\}) = \{b, d\} \neq f^{-1}(A)\) in \((X, \mu_1)\). Hence \(f\) is a contra \(\mu\)-\(\beta\)-generalized \(\alpha\)-continuous mapping, but not a contra \(\mu\)-continuous mapping.

Theorem 3.5: Every contra \(\mu\)-\(\alpha\)-continuous mapping is a contra \(\mu\)-\(\beta\)-generalized \(\alpha\)-continuous mapping in general.

Proof: Let \(f: (X, \mu_1) \rightarrow (Y, \mu_2)\) be a \(\mu\)-\(\alpha\)-contra continuous mapping. Let \(A\) be any \(\mu\)-open set in \((Y, \mu_2)\). Since \(f\) is a \(\mu\)-\(\alpha\)-contra continuous mapping, \(f^{-1}(A)\) is a \(\mu\)-\(\alpha\)-closed set in \((X, \mu_1)\). Since every \(\mu\)-\(\alpha\)-closed set is a \(\mu\)-\(\beta\)-generalized \(\alpha\)-closed set, \(f^{-1}(A)\) is a \(\mu\)-\(\beta\)-generalized \(\alpha\)-closed set in \((X, \mu_1)\). Hence \(f\) is a contra \(\mu\)-\(\beta\)-generalized \(\alpha\)-continuous mapping.

Remark 3.6: A contra \(\mu\)-pre-continuous mapping is not a contra \(\mu\)-\(\beta\)-generalized \(\alpha\)-continuous mapping in general.

Example 3.7: Let \(X = Y = \{a, b, c\}\) with \(\mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}\) and \(\mu_2 = \{\emptyset, \{a\}, Y\}\). Let \(f: (X, \mu_1) \rightarrow (Y, \mu_2)\) be a mapping defined by \(f(a) = a, f(b) = b, f(c) = c\). Now,
\[ \mu^{-}\beta \text{O}(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, c\}, X\}. \]

Let \( A = \{a\} \), then \( A \) is a \( \mu \)-open set in \( (Y, \mu_2) \). Then \( f^{-1}(\{a\}) \) is a \( \mu \)-pre closed set as \( c_{\mu}(i_{\mu}((f^{-1}(A)))) = c_{\mu}(i_{\mu}((a))) = \emptyset \subseteq f^{-1}(A) \), but not a \( \mu \)-generalized \( \alpha \)-closed set as \( \alpha c_{\mu}(f^{-1}(A)) = X \not\subseteq U = \{a, b\} \) in \( (X, \mu_1) \). Hence \( f \) is a contra \( \mu \)-pre-continuous mapping, but not a contra \( \mu \)-generalized \( \alpha \)-continuous mapping.

**Remark 3.8:** A contra \( \mu \)-\( \beta \)-continuous mapping is not a contra \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping in general.

**Example 3.9:** Let \( X = Y = \{a, b, c\} \) with \( \mu_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, c\}, X\} \) and \( \mu_2 = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}, X\} \). Let \( f: (X, \mu_1) \rightarrow (Y, \mu_2) \) be a mapping defined by \( f(a) = a, f(b) = b, f(c) = c \). Now,

\[ \mu^{-}\beta \text{O}(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, c\}, X\}. \]

Let \( A = \{a\} \), then \( A \) is a \( \mu \)-open set in \( (Y, \mu_2) \). Then \( f^{-1}(\{a\}) \) is a \( \mu \)-\( \beta \)-closed set as \( i_{\mu}(c_{\mu}((f^{-1}(A)))) = i_{\mu}(c_{\mu}((a))) = \emptyset \subseteq f^{-1}(A) \), but not \( \mu \)-\( \beta \)-generalized \( \alpha \)-closed set as \( \alpha c_{\mu}(f^{-1}(A)) = X \not\subseteq U = \{a, b\} \) in \( (X, \mu_1) \). Hence \( f \) is a contra \( \mu \)-\( \beta \)-continuous mapping, but not a contra \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping.

In the following diagram, we have provided the relation between various types of contra \( \mu \)-continuous mappings.

\[ \begin{array}{ccc}
\text{contra} & \text{\( \mu \)-continuous} \\
\text{contracontra} & \text{\( \mu \)-\( \alpha \)-continuous} \\
\text{\( \mu \)-\( \beta \)-continuous} & \text{\( \mu \)-\( \beta \)-\( \alpha \)-continuous} \\
\text{contra mu-pre-continuous} \\
\end{array} \]

**Theorem 3.10:** A mapping \( f: (X, \mu_1) \rightarrow (Y, \mu_2) \) is a contra \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping if and only if the inverse image of every \( \mu \)-closed set in \( (Y, \mu_2) \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-open set in \( (X, \mu_1) \).

**Proof:** Necessity: Let \( F \) be a \( \mu \)-closed set in \( (Y, \mu_2) \). Then \( Y-F \) is a \( \mu \)-open in \( (Y, \mu_2) \). Then \( f^{-1}(Y-F) \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-closed set in \( (X, \mu_1) \), by hypothesis. Since \( f^{-1}(Y-F) = X - f^{-1}(F) \), Hence \( f^{-1}(F) \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-open set in \( (X, \mu_1) \).

**Sufficiency:** Let \( F \) be a \( \mu \)-open set in \( (Y, \mu_2) \). Then \( Y-F \) is a \( \mu \)-closed in \( (Y, \mu_2) \). By hypothesis, \( f^{-1}(Y-F) \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-open set in \( (X, \mu_1) \). Since \( f^{-1}(Y-F) = X - f^{-1}(F) \), Hence \( f^{-1}(F) \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-open set in \( (X, \mu_1) \). Hence \( f \) is a contra \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping.

**Theorem 3.11:** Let \( f: (X, \mu_1) \rightarrow (Y, \mu_2) \) be a mapping and let \( f^{-1}(V) \) be a \( \mu \)-open set in \( (X, \mu_2) \) for every closed \( V \) set in \( (Y, \mu_2) \). Then \( f \) is a contra \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping.

**Proof:** Let \( V \) be a \( \mu \)-closed set in \( (Y, \mu_2) \). Then \( f^{-1}(V) \) be a \( \mu \)-open set in \( (X, \mu_1) \), by hypothesis. Since every \( \mu \)-open set is \( \mu \)-\( \beta \)-generalized \( \alpha \)-open set in \( X \). Hence \( f^{-1}(V) \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-open set in \( (X, \mu_1) \). Hence \( f \) is a contra \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping.

**Theorem 3.12:** If \( f: (X, \mu_1) \rightarrow (Y, \mu_2) \) is a contra \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping and \( g: (Y, \mu_2) \rightarrow (Z, \mu_3) \) is a \( \mu \)-continuous mapping then \( g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3) \) is a contra \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping.

**Proof:** Let \( V \) be any \( \mu \)-open set in \( (Z, \mu_3) \). Then \( g^{-1}(V) \) is a \( \mu \)-open set in \( (Y, \mu_2) \), since \( g \) is a \( \mu \)-continuous mapping. Since \( f \) is a contra \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping, \( (g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-closed set in \( (X, \mu_1) \). Therefore \( g \circ f \) is a contra \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping.

**Theorem 3.13:** If \( f: (X, \mu_1) \rightarrow (Y, \mu_2) \) is a contra \( \mu \)-continuous mapping and \( g: (Y, \mu_2) \rightarrow (Z, \mu_3) \) is a contra \( \mu \)-continuous mapping then \( g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3) \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping.

**Proof:** Let \( V \) be any \( \mu \)-open set in \( (Z, \mu_3) \). Since \( g \) is a \( \mu \)-continuous mapping, \( g^{-1}(V) \) is a \( \mu \)-closed set in \( (Y, \mu_2) \). Since \( f \) is a contra \( \mu \)-continuous mapping, \( (g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) \) is a \( \mu \)-open set in \( (X, \mu_1) \). Since every \( \mu \)-open set is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-open set, \( (g \circ f)^{-1}(V) \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-open set in \( (X, \mu_1) \). Therefore \( g \circ f \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping.

**Theorem 3.14:** If \( f: (X, \mu_1) \rightarrow (Y, \mu_2) \) is a contra \( \mu \)-\( \alpha \)-continuous mapping and \( g: (Y, \mu_2) \rightarrow (Z, \mu_3) \) is a contra \( \mu \)-continuous mapping then \( g \circ f: (X, \mu_1) \rightarrow (Z, \mu_3) \) is a \( \mu \)-\( \beta \)-generalized \( \alpha \)-continuous mapping.
Proof: Let V be any μ-closed set in (Z, μ₁). Since g is a μ-contra continuous mapping, g⁻¹(V) is a μ-open set in (Y, μ₂). Since f is a μ-α-contra continuous mapping, (g \circ f)⁻¹(V) = f⁻¹(g⁻¹(V)) is a μ-α-closed set in (X, μ₁). Since every μ-α-closed set is a μ-β-generalized α-closed set, (g \circ f)⁻¹(V) is a μ-β-generalized α-closed set in (X, μ₁). Therefore g \circ f is a μ-β-generalized α-continuous mapping.

Theorem 3.15: If f: (X, μ₁) \rightarrow (Y, μ₂) is a μ-continuous mapping and g: (Y, μ₂) \rightarrow (Z, μ₃) is a contra μ-continuous mapping then g \circ f: (X, μ₁) \rightarrow (Z, μ₃) is a contra μ-β-generalized α-continuous mapping.

Proof: Let V be any μ-open set in (Z, μ₃). Since g is a contra μ-continuous mapping, g⁻¹(V) is a μ-closed set in (Y, μ₂). Since f is a μ-α-contra continuous mapping, (g \circ f)⁻¹(V) = f⁻¹(g⁻¹(V)) is a μ-closed set in (X, μ₁). Since every μ-closed set is a μ-β-generalized α-closed set, (g \circ f)⁻¹(V) is a μ-β-generalized α-closed set. Therefore g \circ f is a contra μ-β-generalized α-continuous mapping.

4. ALMOST CONTRA μ-β-GENERALIZED α-CONTINUOUS MAPPINGS

In this section we have introduced almost contra μ-β-generalized α-continuous mapping in generalized topological spaces and studied some of their basic properties.

Definition 4.1: A mapping f: (X, μ₁) \rightarrow (Y, μ₂) is called an almost contra μ-β-generalized α-continuous mapping if f⁻¹(A) is a μ-β-generalized α-closed set in (X, μ₁) for each μ-regular open set A in (Y, μ₂).

Example 4.2: Let X = Y = \{a, b, c\} with μ₁ = \{Ø, \{a\}, \{a, b\}, \{a, b, c\}, Y\} and μ₂ = \{Ø, \{a\}, \{a, b\}, \{a, b, c\}, Y\}. Let f: (X, μ₁) \rightarrow (Y, μ₂) be a mapping defined by f(a) = a, f(b) = b, f(c) = c. Now, μ-βO(X) = \{Ø, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, c\}, X\}.

Let A = \{c\}, then A is a μ-regular open set in (Y, μ₂). Then f⁻¹(\{c\}) is a μ-β-generalized α-closed set in (X, μ₁). Hence f is an almost contra μ-β-generalized α-continuous mapping.

Theorem 4.3: Every almost contra μ-continuous mapping is an almost contra μ-β-generalized α-continuous mapping but not conversely.

Proof: Let f: (X, μ₁) \rightarrow (Y, μ₂) be an almost contra μ-continuous mapping. Let A be any μ-regular open set in (Y, μ₂). Since f is almost contra μ-continuous mapping, f⁻¹(A) is a μ-closed set in (X, μ₁). Since every μ-closed set is a μ-β-generalized α-closed set, f⁻¹(A) is a μ-β-generalized α-closed set in (X, μ₁). Hence f is an almost contra μ-β-generalized α-continuous mapping.

Example 4.4: Let X = Y = \{a, b, c, d\} with μ₁ = \{Ø, \{a\}, \{a, c\}, X\} and μ₂ = \{Ø, \{d\}, \{a, b, c\}, Y\}. Let f: (X, μ₁) \rightarrow (Y, μ₂) be a mapping defined by f(a) = a, f(b) = b, f(c) = c, f(d) = d. Now, μ-βO(X) = \{Ø, \{a\}, \{c\}, \{a, b\}, \{a, d\}, \{b, c\}, \{a, c, d\}\}.

Let A = \{d\}, then A is a μ-regular open set in (Y, μ₂). Then f⁻¹(\{d\}) is a μ-β-generalized α-closed set, but not μ-closed as cμ(f⁻¹(\{d\})) = cμ(\{b, d\}) = \{b, d\} ≠ f⁻¹(A) in (X, μ₁). Hence f is an almost contra μ-β-generalized α-continuous mapping, but not almost contra μ-continuous mapping.

Theorem 4.5: Every almost contra μ-α-continuous mapping is an almost contra μ-β-generalized α-continuous mapping in general.

Proof: Let f: (X, μ₁) \rightarrow (Y, μ₂) be an almost contra μ-α-continuous mapping. Let A be any μ-regular open set in (Y, μ₂). Since f is an almost contra μ-α-continuous mapping, f⁻¹(A) is a μ-α-closed set in (X, μ₁). Since every μ-α-closed set is a μ-β-generalized α-closed set, f⁻¹(A) is a μ-β-generalized α-closed set in (X, μ₁). Hence f is an almost contra μ-β-generalized α-continuous mapping.

Remark 4.6: An almost contra μ-β-pre-continuous mapping is not an almost contra μ-β-generalized α-continuous mapping in general.

Example 4.7: Let X = Y = \{a, b, c\} with μ₁ = \{Ø, \{a\}, \{a, b\}, \{a, b, c\}, Y\} and μ₂ = \{Ø, \{a\}, \{a, b\}, \{a, b, c\}, Y\}. Let f: (X, μ₁) \rightarrow (Y, μ₂) be a mapping defined by f(a) = a, f(b) = b, f(c) = c. Now, μ-βO(X) = \{Ø, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, c\}, X\}.

Let A = \{a\}, then A is a μ-regular open set in (Y, μ₂). Then f⁻¹(\{a\}) is a μ-pre closed as cμ(\{a\}) = \{Ø \subseteq f⁻¹(\{a\})\}, but not μ-β-generalized α-closed set as αcμ(f⁻¹(\{a\})) = X ≠ U = \{a, b\} in (X, μ₁). Hence f is an almost contra μ-continuous mapping.
Remark 4.8: An almost contra $\mu$-$\beta$-continuous mapping is not an almost contra $\mu$-$\beta$-generalized $\alpha$-continuous mapping in general.

Example 4.9: Let $X = Y = \{a, b, c\}$ with $\mu_1 = \{\emptyset, \{a, b\}, X\}$ and $\mu_2 = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Let $f: (X, \mu_1) \to (Y, \mu_2)$ be a mapping defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Now,

$$\mu_\beta O(X) = \{\emptyset, \{a\}, \{b, c\}, \{a, c\}, X\}.$$

Let $A = \{a\}$, then $A$ is a $\mu$-regular open set in $(Y, \mu_2)$. Then $f^{-1}(\{a\})$ is a $\mu$-$\beta$-closed set as $i_\mu(c_\mu(i_\mu(f^{-1}(A)))) = i_\mu(c_\mu((a)))) = \emptyset \subseteq f^{-1}(A)$, but not a $\mu$-$\beta$-generalized $\alpha$-closed set as $\alpha c_\mu(f^{-1}(A)) = X \nsubseteq U = \{a, b\}$ in $(X, \mu_1)$. Hence $f$ is an almost contra $\mu$-$\beta$-continuous mapping, but not almost contra $\mu$-$\beta$-generalized $\alpha$-continuous mapping.

In the following diagram, we have provided the relation between various types of almost contra $\mu$-continuous mappings.

![Diagram showing the relationship between different types of mappings]

**Theorem 4.10:** A mapping $f: (X, \mu_1) \to (Y, \mu_2)$ is an almost contra $\mu$-$\beta$-generalized $\alpha$-continuous mapping if and only if the inverse image of every $\mu$-regular closed set in $(Y, \mu_2)$ is a $\mu$-$\beta$-generalized $\alpha$-open set in $(X, \mu_1)$.

**Proof:**

**Necessity:** Let $F$ be a $\mu$-regular closed set in $(Y, \mu_2)$. Then $F = f^{-1}(U)$ for some $\mu$-open set $U$ in $(Y, \mu_2)$. Since $f$ is a $\mu$-$\beta$-generalized $\alpha$-continuous mapping, $f^{-1}(U)$ is a $\mu$-$\beta$-generalized $\alpha$-closed set in $(X, \mu_1)$. Therefore $f^{-1}(F)$ is a $\mu$-$\beta$-generalized $\alpha$-open set in $(X, \mu_1)$.

**Sufficiency:** Let $F$ be a $\mu$-regular open set in $(Y, \mu_2)$. Then $F = f^{-1}(U)$ for some $\mu$-closed set $U$ in $(Y, \mu_2)$. By hypothesis, $f^{-1}(U)$ is a $\mu$-$\beta$-generalized $\alpha$-open set in $(X, \mu_1)$. Hence $f^{-1}(F)$ is a $\mu$-$\beta$-generalized $\alpha$-closed set in $(X, \mu_1)$. Hence $f$ is an almost contra $\mu$-$\beta$-generalized $\alpha$-continuous mapping.

**Theorem 4.11:** If $f: (X, \mu_1) \to (Y, \mu_2)$ is a $\mu$-continuous mapping and $g: (Y, \mu_2) \to (Z, \mu_3)$ is an almost contra $\mu$-continuous mapping then $g \circ f: (X, \mu_1) \to (Z, \mu_3)$ is an almost contra $\mu$-$\beta$-generalized $\alpha$-continuous mapping.

**Proof:** Let $V$ be any $\mu$-regular open set in $(Z, \mu_3)$. Since $g$ is an almost contra $\mu$-continuous mapping, $g^{-1}(V)$ is a $\mu$-closed set in $(Y, \mu_2)$. Since $f$ is a $\mu$-continuous mapping, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is a $\mu$-closed in $(X, \mu_1)$. Hence $f$ is an almost contra $\mu$-$\beta$-generalized $\alpha$-continuous mapping.

**Theorem 4.12:** If $f: (X, \mu_1) \to (Y, \mu_2)$ is a $\mu$-$\alpha$-continuous mapping and $g: (Y, \mu_2) \to (Z, \mu_3)$ is an almost contra $\mu$-continuous mapping then $g \circ f: (X, \mu_1) \to (Z, \mu_3)$ is a contra $\mu$-$\beta$-generalized $\alpha$-continuous mapping.

**Reference**

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